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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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CONTENTS

Papers: PAGE

Civil Engineering Features of the Hell Gate Station.
By ERNEST M. VAN NORDEN and GEORGE A. HUGHES, MEMBERS, AM. SOC. C. E. 337

The Civil Engineering Features of the Kearny Power Station.
By ROMAN VON FABRICE, ASSOC. M. AM. SOC. C. E. 366

Discussions:

Aeroplane Topographic Surveys.
By GEORGE T. BERGEN, M. AM. SOC. C. E. 419

Field Procedure of Adjusting the Great Circle Line to the Rhumb Line.
By N. B. SWEITZER, ASSOC. M. AM. SOC. C. E. 427

Snow Removal Problems of Organization and Operation.
By GEORGE E. HAMLIN, M. AM. SOC. C. E. 436

Straight Line Plotting of Skew Frequency Data.
By MESSRS. E. P. GOODRICH, ARNE FISHER, and ALLEN HAZEN 438

Analysis of Continuous Concrete Arch Systems.
By HARDY CROSS, M. AM. SOC. C. E. 471

Distribution of Reinforcing Steel in Concrete Beams and Slabs.
By JOHN J. GOULD, ASSOC. M. AM. SOC. C. E. 480

Grit Chamber Practice: A Symposium.
By R. S. RANKIN, ESQ. 483

Producing Concrete of Uniform Quality.
By JOHN G. AHLERS, M. AM. SOC. C. E. 486

Water-Ratio Specification for Concrete.
By JOHN G. AHLERS, M. AM. SOC. C. E. 488

Unit Stresses in Structural Materials.
By CLEMENT E. CHASE, M. AM. SOC. C. E. 490

Water-Proof Masonry Dams.
By MESSRS. THOMAS H. WIGGIN, JEROME W. HOWE, W. H. R. NIMMO, and E. W. LANE 497

Memoirs:

OSWALD HERBERT ERNST, M. AM. SOC. C. E. 505
HERBERT CLARK FELTON, M. AM. SOC. C. E. 507
SAMUEL BROWNLEE FISHER, M. AM. SOC. C. E. 508
HARRY ADOLPH OESTERLE, JR., ASSOC. M. AM. SOC. C. E. 510

For Index to all Papers, the discussion of which is current in *Proceedings*, see the second page of the cover.

CIVIL ENGINEERING FEATURES OF THE HELL GATE STATION*

By Ernest M. Van Norden,† and George A. Hughes,‡
Members, Am. Soc. C. E.

The Hell Gate Station of The United Electric Light and Power Company, located on the East River between 132d and 134th Streets, New York, N. Y., has been well described in the technical press and also in publications issued from time to time by its owners. It is very natural in such publicity for the author to confine himself to a description of the mechanical and electrical equipment and to neglect much of the detail pertaining to the structure. The problems involving civil engineering, within the commonly accepted meaning of this term, are usually lightly touched upon, although these problems may be difficult of solution, intricate in design, and expensive in execution.

The Hell Gate Station involved many problems in this class, some of which are not found in other power station designs; and for their successful solution much credit is due the designers and contractors. It is the purpose of this paper to describe the more important civil engineering features of this station, introducing only such mechanical and electrical details as may be required to make a complete picture.

Three factors controlled the selection of the site:

First.—Proximity to the center of gravity of the load.

Second.—Real estate values.

Third.—Water-front facilities for receiving coal and supplying circulating water for the condensers.

The station was designed to supply electric energy to Queens Borough and Westchester County, as well as to the Boroughs of Manhattan and The Bronx, and from the standpoint of center of gravity of the load the site was well chosen. Even with considerable increase in the load the center of gravity would still remain about the same.

From the standpoint of real estate values the Company was fortunate in being able to secure this property at a reasonable price. The number of water-front properties available in the City of New York that could be used for power-station development are few, and the question of cost might be regarded as unimportant, although in this particular case the relatively low cost tended to make a quicker decision as to location.

* Presented at the meeting of the Power Division, New York, N. Y., January 21, 1926.
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The Company succeeded in securing from the city the exclusive privilege of mooring at the foot of 134th Street. The entire length of water-front, therefore, ran from the north side of 132d Street to the north side of 134th Street, a distance of about 500 ft. It was planned to receive coal at this station in ocean-going ships capable of carrying 10 000 tons with a draft of 28 ft., but the capacity of the boats which so far have entered this service rarely exceeds 6 000 tons, with a draft of 24 ft. To accommodate the large ships safely it was necessary to obtain 35 ft. of water alongside the wharf.

An aeroplane photograph looking toward the northeast (Fig. 1) illustrates the excellent water-front advantages enjoyed by this station. The area immediately south of 132d Street, shown in the foreground, is to be developed as a freight yard for the New York, New Haven and Hartford Railroad Company which plans to build a bulkhead at the easterly end for transferring water and railroad freight. At the time this photograph was taken the station was not entirely completed, the view showing only three stacks of a total of four.

The survey to determine the elevations of the surface and the bed-rock had to be carried beyond the low-water mark. The wash borings that were made, partly on land and partly from floats, proved to be very accurate, making it possible to contract on a lump-sum basis for the removal of the over-burden and the rock down to the levels required.

As shown in the topographical survey of the rock surface (Fig. 3), the contour lines, assuming the over-burden to be removed, indicate that the rock slopes rapidly down offshore. On this map is indicated the main outline of the station structure.

The turbo-generator room was located parallel with the water-front so that the water for the condensers could be pumped directly from the river, thereby avoiding the construction of a long intake tunnel. The discharge tunnel for the condensing water was also located parallel with the water-front and as close to the river wall of the station as possible. The absence of an intake tunnel for the condensing water renders this station unique among power stations in the City of New York.

The entire rock excavation, therefore, was confined to an area very close to the water-front and was done by under-water blasting. It was necessary to go to certain depths for the discharge tunnel and for the suction pipes and screens. It was also necessary to provide sufficient depth of water for ocean-going ships along the entire length of the wharf.

Fig. 4 is a cross-section of the station showing the cut required in the rock and also the relative position of the sea-wall supporting the easterly wall of the station and the discharge tunnel and wharf.

The contractor first dredged the site to remove the over-burden and then used two drill boats especially constructed for submarine drilling. The size of the drill bits was $4\frac{3}{4}$ in., making holes approximately 5 in. in diameter. These holes were drilled on 5-ft. centers in both directions over the entire area and were carried down to 5 or 6 ft. below the finished rock grade.

The operation of drilling was as follows: The drill boat was located by shore rangers and held in position by lines run out to anchors, by lines to the

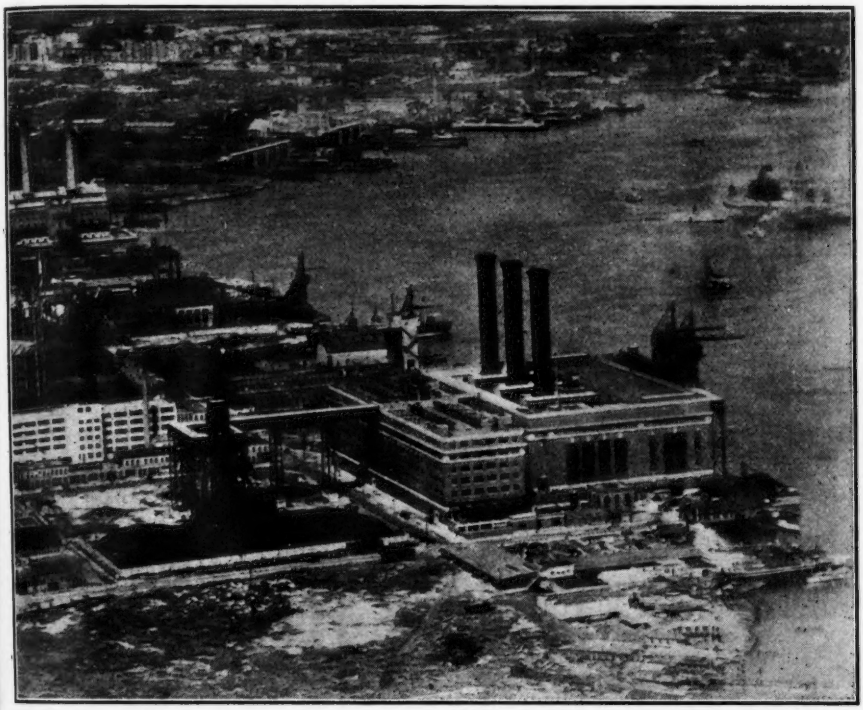


FIG. 1.—AEROPLANE VIEW, SHOWING WATER-FRONT ADVANTAGES OF THE HELL GATE STATION.

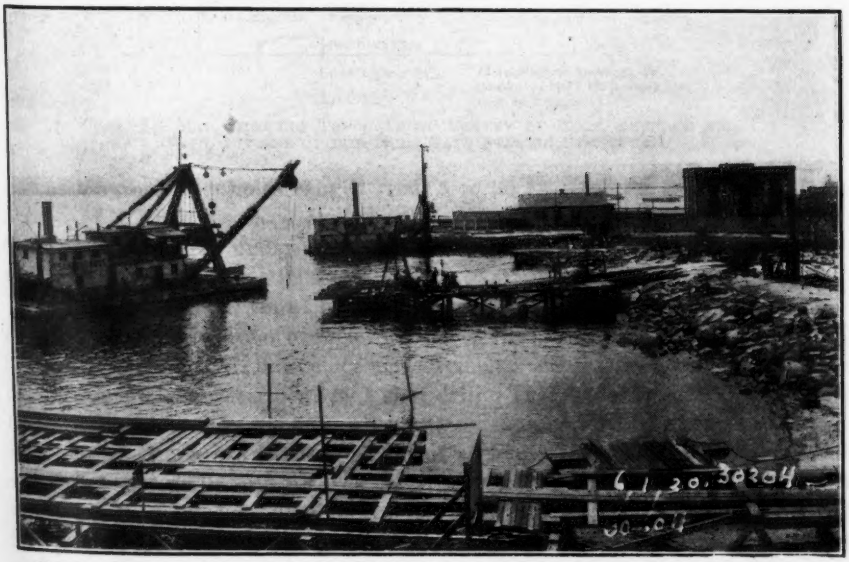


FIG. 2.—VIEW OF DRILL BOAT IN POSITION AND DREDGE IN OPERATION, HELL GATE STATION.

shore, and also by spuds on the boat resting on the river bed. A drill pipe was dropped down until it rested on the bottom, and the drill rod was dropped through the pipe. The drill was operated by steam and was fed down as the rock was drilled. After the hole was finished it was loaded with dynamite through the drill pipe and the wire brought up and attached to the boat. Every day all the wires were connected, the boat pulled out stream beyond danger, and these holes were fired. The boat then took up a new position and commenced drilling again. After the area had been drilled over it was dredged off and, if soundings indicated high spots, these were re-drilled and the same procedure was repeated.

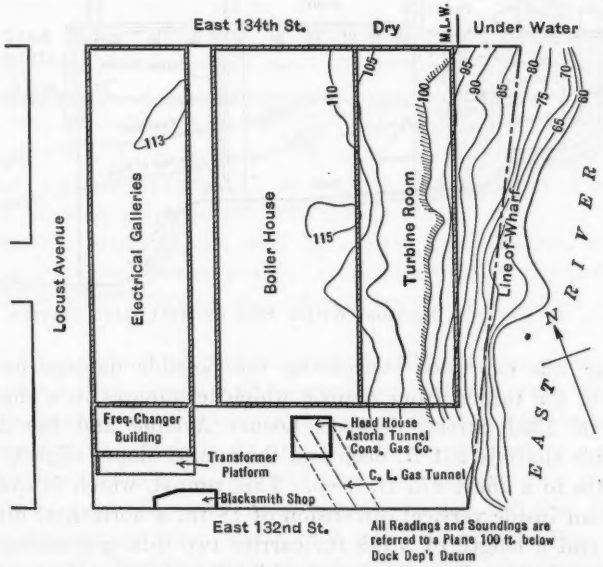


FIG. 3.—MAP SHOWING TOPOGRAPHIC SURVEY OF ROCK SURFACE AND MAIN OUTLINE OF THE HELL GATE STATION STRUCTURE.

The length of the holes ranged from 6 to 20 ft., with an average of 11 ft. A total of 1 776 holes were drilled. The average rate of drilling was about 60 ft. per day. This under-water work was started in March, 1920, and was completed in September, 1921.

There was 2 000 cu. yd. of over-burden dredged before drilling of rock was started. The total volume of rock removed by the dredge was 23 600 cu. yd. and, in the dry, 13 400 cu. yd., both place measurement. An attempt was made to have the work in shallow water done by a shot drill. This was started about opposite 133d Street. A total of ten holes about 20 ft. deep were drilled when this method was abandoned because the soft strata of rock would cause the core to break and wedge the drills and also caused them to lose their shot. These holes were 6 in. in diameter. The contractor then tried a wagon drill near 134th Street in shallow water, but progress was so slow that it was abandoned. It was the custom to bring the drill boat in at high tide and work offshore at low tide. Fig. 2 looking south shows the drill boat in position at the south

end of the site and also the dredge in operation. The temporary pier shown in the center of the photograph was constructed to accommodate the shot drilling.

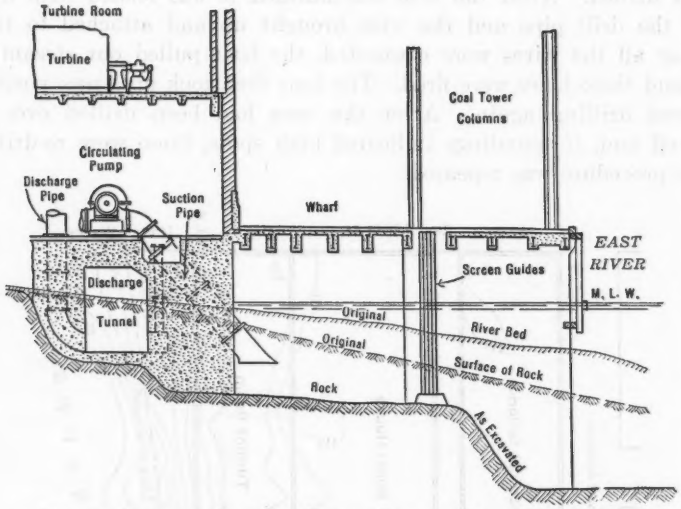


FIG. 4.—VERTICAL SECTION, WHARF SIDE OF HELL GATE STATION.

Some fear was expressed concerning the possible damage by reason of the blasting to the tunnel from Astoria which terminates in a shaft immediately north of 132d Street, between Locust Avenue and the East River (Fig. 3). This shaft is 231 ft. deep and the tunnel slopes slightly downward toward Astoria to a shaft 275 ft. deep. This tunnel, which is oval in cross-section, with an inside vertical dimension of 18 ft., a horizontal dimension of 16 ft. 9 in., and a length of 5 168 ft., carries two 6-ft. gas mains and some smaller pipes and conduits. During the blasting an inspector was stationed in the tunnel for purposes of observation and as a precaution the charges were made lighter at the south end of the excavation near the tunnel. The blasts were barely perceptible in the tunnel and no damage occurred.

To save the cost and delay of a coffer-dam the construction of the river-wall footing down to bed-rock at a depth of approximately 16 ft. below mean low water was started immediately on the completion of the rock excavation. This footing was designed with returns inshore so as to take the part of a coffer-dam and permit the construction of a discharge tunnel in the open. The foundation of the river wall was made of concrete deposited by tremie. For convenience in construction it was divided into sections about 14 ft. long, alternate sections being constructed in the first operation. The second operation filled in between the alternate sections thus completing the continuous wall with its return.

The concrete sea-wall is trapezoidal in cross-section with an average depth of 28.5 ft., and a base 12 ft. wide. Alternate sections about 14 ft. long were first poured, with keys formed in the ends. The forms, built on the ground

in advance, in the shape of an open box, were of 2 by 10-in. planks laid in a horizontal direction on all four sides. On the ends these were reinforced with 6 by 6-in. timbers and tie-rods 1 in. in diameter and on the sides the reinforcement consisted of 12 and 15-in. steel channels with similar tie-rods. The rods were run through pipe sleeves so that they could be easily removed after the concrete had set. After the forms were lowered into place by weighting them with reinforcing bars, the voids between the bottom of the box and the rock were filled in with sand bags placed by divers. Then the bottom was sealed with concrete poured by the tremie method, the sea water pumped out, and the remainder of the wall poured in the dry.

Fig. 5 shows one section of the concrete wall in place with the forms stripped and an adjacent form being lowered into place. This view is taken looking north along the line of the sea-wall, showing the movable plant of the contractor at the left. After the forms were placed in position divers sealed the spaces between the bottom of the form and irregular rock surfaces.

Fig. 6 was taken from the south end of the rock showing the opening for the discharge tunnel through the sea-wall in the foreground and also the operation of shooting concrete into the tremie funnel.

Provision was made in the wall to receive the cast-iron suction pipes which run from the condenser pumps in the basement of the turbo-generator room at a slope of 45° through the concrete wall to a point 5 ft. below mean low water.

After the completion of the concrete footing of the river wall of the station the water back of the wall was pumped out and work was started on the discharge tunnel and the basement floor of the turbo-generator room, the finished elevation of which is 11 ft. above mean low water. The sea-wall proved to be exceptionally water-tight and the bed-rock was free from any open veins. To keep the excavation back of the sea-wall free from water a single 8-in. centrifugal pump was found to be adequate. Fig. 7 shows the rear of the sea-wall after being unwatered. The 52-in. cast-iron suction pipes are shown concreted in place and the concrete bottom of the discharge tunnel is also shown resting directly on the bed-rock. In the foreground the strike of the rock is seen to run parallel with the sea-wall.

Owing to experience at other stations in the disintegration of concrete in discharge tunnels special care was taken to make the inside surfaces of the Hell Gate discharge tunnel of a smooth, hard, and impervious concrete. To use a similar high-grade concrete for back-filling under the entire basement would have been unnecessary waste and, therefore, a method was devised to place high-grade concrete at least 12 in. thick next to the discharge tunnel form and at the same time, lean concrete for the large mass. Fig. 8 shows the tunnel in an advanced state of construction. In addition to the suction pipes shown in the other photographs it also shows on the left-hand side the 42-in. cast-iron discharge pipes which were embedded in the concrete fill in the basement of the turbo-generator room terminating in an elbow at the bottom of the discharge tunnel. This tunnel is 11½ ft. wide by 15½ ft. high, running the full length of the turbo-generator room, the bottom elevation being 10 ft. below mean low water.

The "mixed aggregate" of sand and gravel used on this work was delivered in barge load lots and unloaded with a clam-shell bucket. In depositing this material in the storage piles the separation of the sand and gravel, particularly when the piling was done in cones, was so marked that re-apportioning of the ingredients was necessary to insure uniformity of the mixture for the special high-grade concrete used in the lining of the discharge tunnel and the reinforced concrete work.

The wharf is constructed of reinforced concrete slabs spanning between deep Bethlehem I-beams protected with a 4½-in. covering of concrete. The steel framing is supported by rectangular concrete piers, 6 ft. wide and 31 ft. long, reinforced with vertical rods and spaced 30 ft. 3 in. on centers. In several places cylindrical piers were used, 6 ft. in diameter and 46 ft. deep, and reinforced with thirty-two vertical rods, 1⅛ in. in diameter, and hoops, 12 in. on centers. The bottom of the pier is bonded into the rock with three dowels, 3½ in. in diameter. The deck of the wharf is finished with 4½-in. paving brick laid in a 1½-in. sand cushion.

The river water passes through steel screens before entering the intake basins. Each basin has an opening 24 ft. wide and 20 ft. high and is protected by four removable screens set in vertical bars spaced about 1 in. on centers and mounted in 4-in. channel frames. All the concrete below the water line was deposited through tremie pipes in forms sealed at the bottom as in the case of the sea-wall. Fig. 9 shows the wharf construction with the forms for the girder covering and deck slabs started. Fig. 10 is a "close-up" of the screen construction, showing the opening in the deck of the wharf to give access to the screens and also the base of the steel work for the coal trestle.

To provide for the ashes, which are discharged hydraulically through pipes to the water-front, a concrete bin was constructed within the lines of the wharf into which the ashes are deposited and from which they are removed by a clam-shell bucket. The hoisting apparatus (Fig. 11) is arranged so that the ashes may be deposited in barges alongside or in trucks.

The steel trestle on the wharf, which is 107 ft. high and supports two traveling coal towers is also shown in Fig. 11. In this trestle construction is shown the crane beams for removing the ashes from the ash-pit so arranged as to permit ashes to be deposited in barges or in motor trucks.

On top of this trestle is carried a system of double tracks which parallel the bulkhead about at the level of the roof of the station. The tracks continue across bridges connecting the trestle with the turbine room, and another bridge spans the court between the boiler-house and control galleries. Where the cars cross the turbine room, the tracks are supported on the bottom chord of the roof trusses which have a span of 98 ft. West of the control galleries, the tracks are again carried on a steel trestle spanning Locust Avenue and extending 175 ft. to the center of the two city blocks used for coal storage (Figs. 12 and 13). This structure has an expansion joint at the control galleries and another at the rigid tower construction, and the three intermediate truss spans are supported by two flexible towers. The wind pressure on this trestle was assumed at 24 lb. per sq. ft., and the customary fiber stresses for

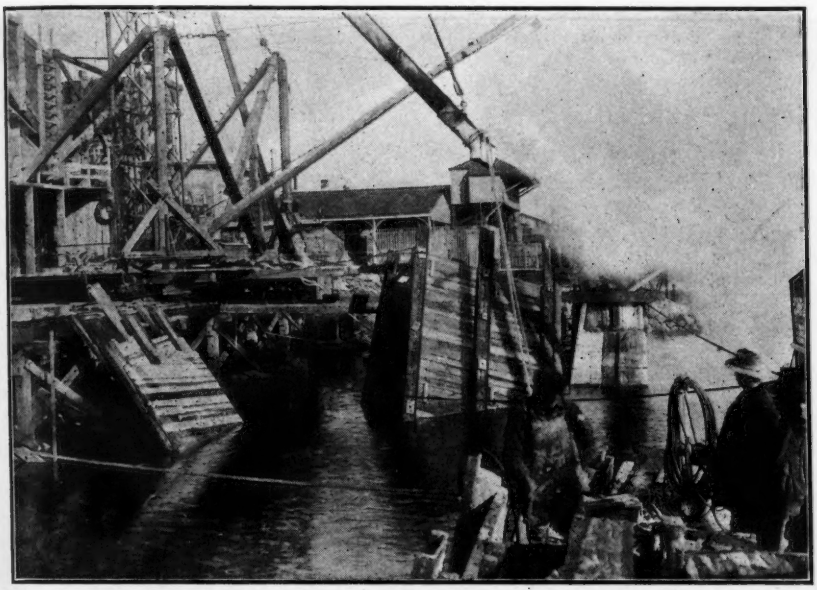


FIG. 5.—HELL GATE STATION: VIEW SHOWING SECTION OF CONCRETE WALL IN PLACE, WITH FORMS STRIPPED AND ADJACENT FORM BEING LOWERED.

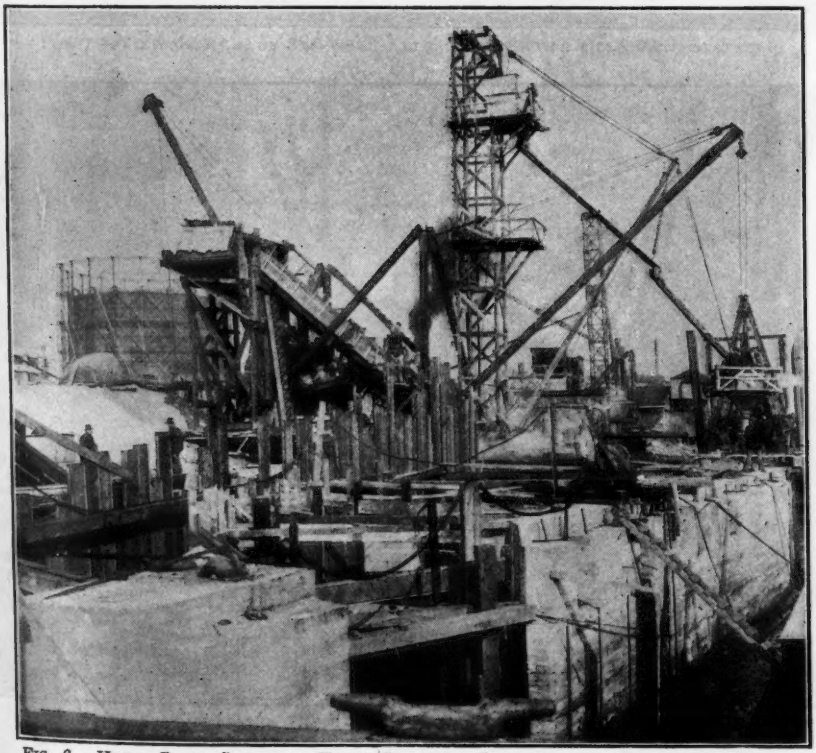


FIG. 6.—HELL GATE STATION: VIEW SHOWING OPENING FOR DISCHARGE TUNNEL THROUGH THE SEA-WALL, AND OPERATION OF SHOOTING CONCRETE INTO TREMIE FUNNEL.

FIG. 7.—VIEW OF REAR OF SEA-WALL AFTER UNWATERING, HELL GATE STATION.

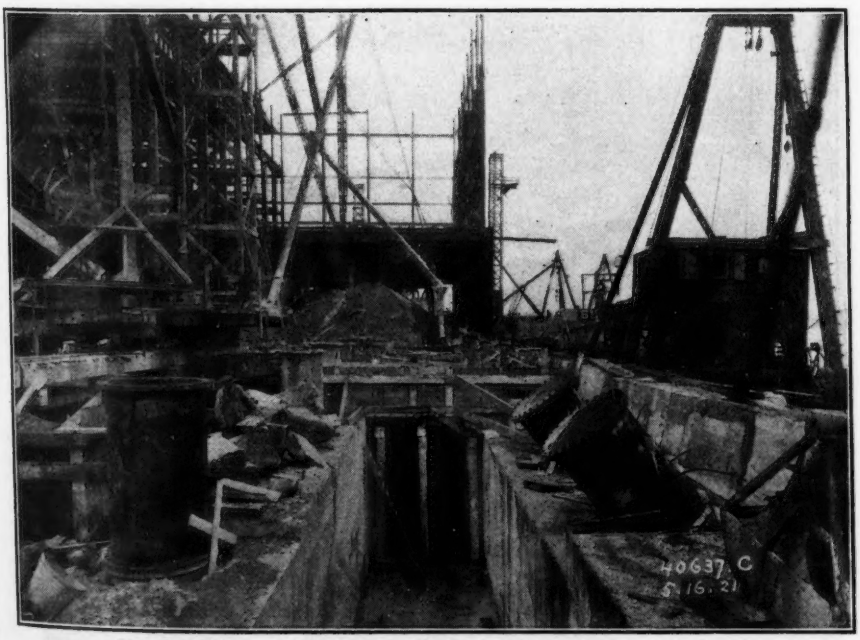


FIG. 8.—DISCHARGE TUNNEL IN ADVANCED STATE OF CONSTRUCTION, HELL GATE STATION.

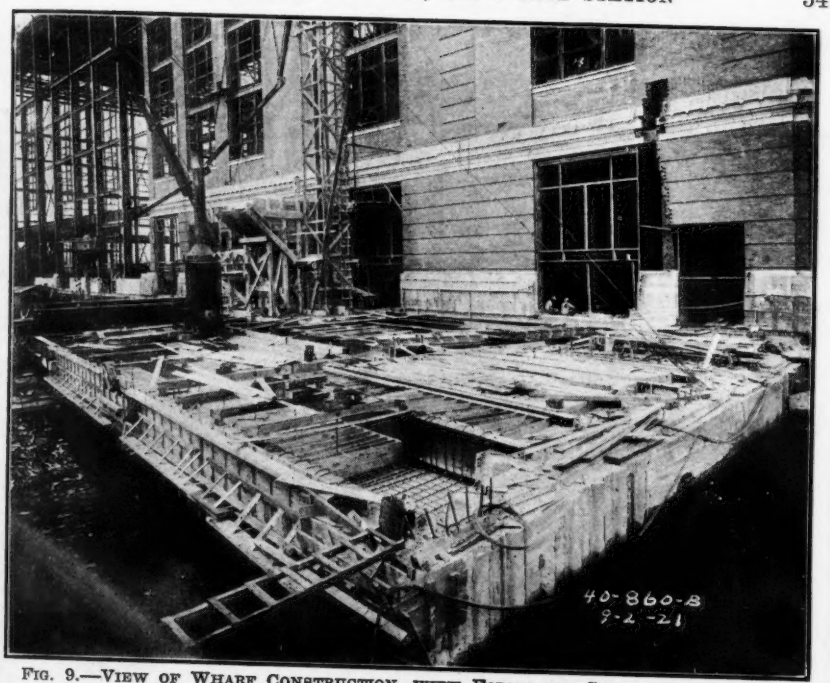


FIG. 9.—VIEW OF WHARF CONSTRUCTION, WITH FORMS FOR GIRDER COVERING AND DECK SLABS STARTED, HELL GATE STATION.

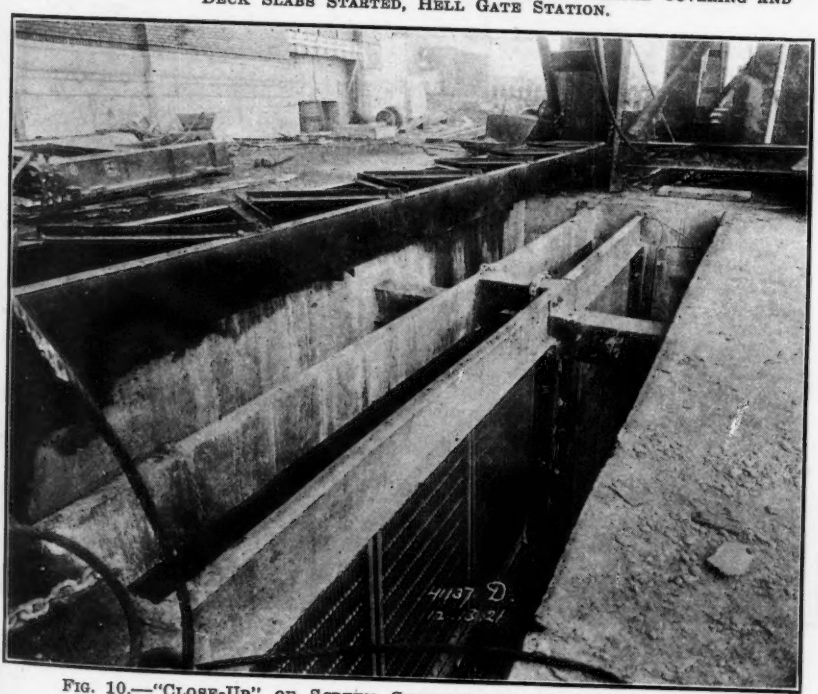
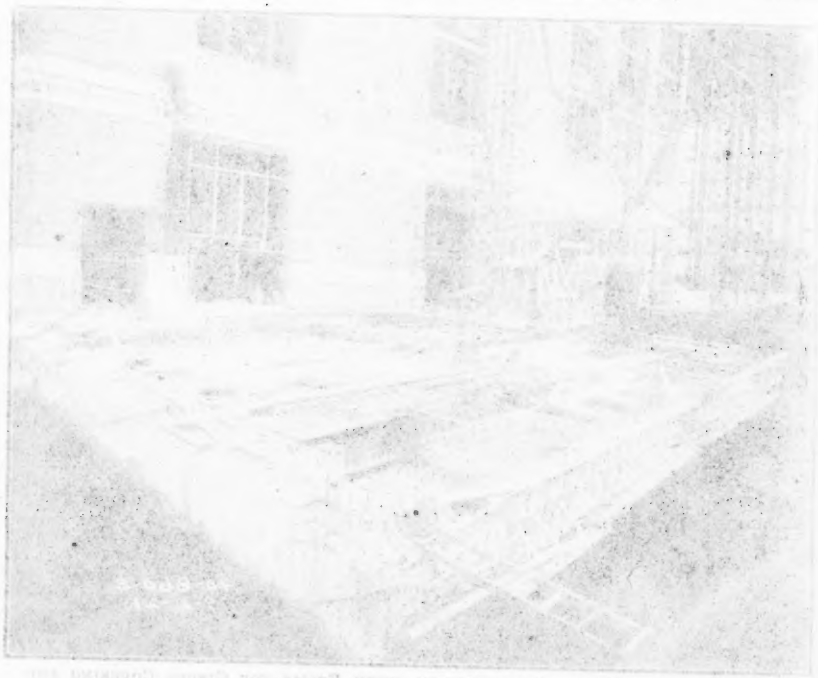


FIG. 10.—"CLOSE-UP" OF SCREEN CONSTRUCTION, HELL GATE STATION.



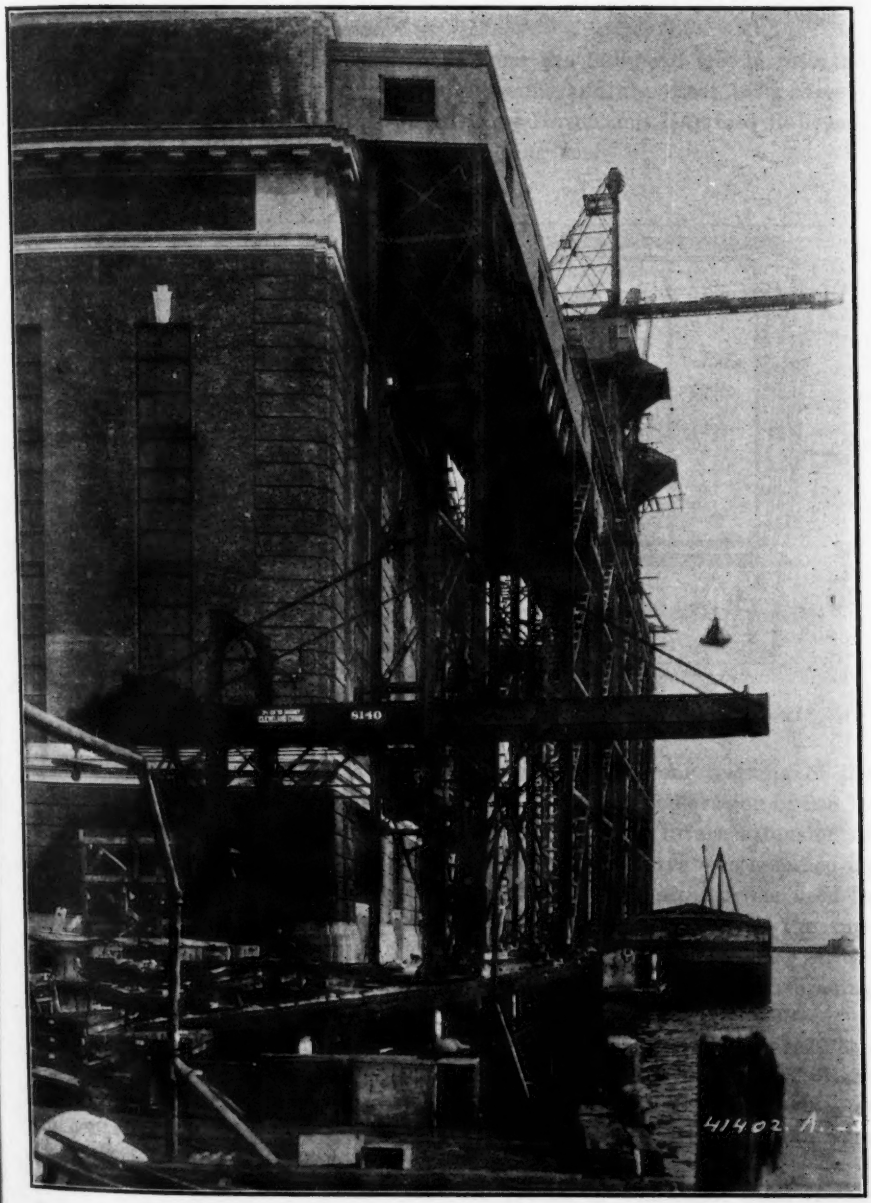
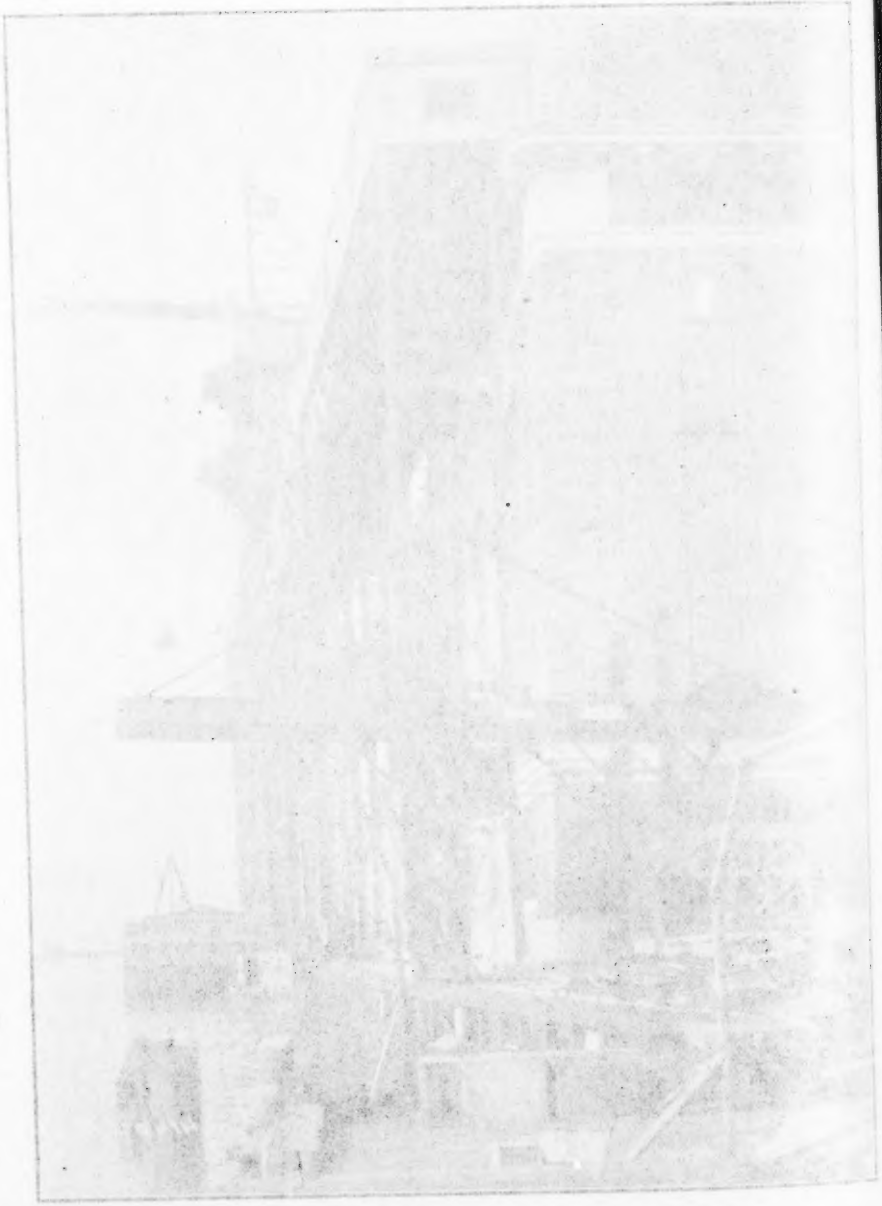


FIG. 11.—VIEW OF HOISTING APPARATUS, HELL GATE STATION.



static loads were used in designing the steel work. The cars deliver the coal to receiving hoppers suspended from the track floor and from there it is carried through steel chutes, 65 ft. long, and piled 35 ft. high in the storage yard for distribution by a drag scraper.

In Fig. 12 are shown the coal tracks from the bulkhead trestle into the storage yard in the rear, crossing Locust Avenue and also East 133d Street; also the track siding from the New York, New Haven and Hartford Railroad, which serves as an auxiliary means of supplying coal.

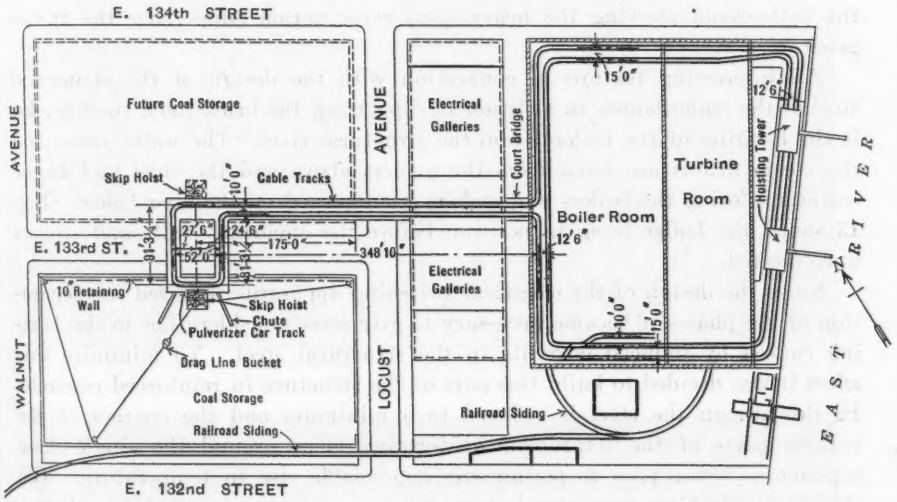


FIG. 12.—PLAN OF STATION AND COAL STORAGE YARD, HELL GATE STATION.

Four self-supporting steel stacks are carried on the roof framing of the boiler-house. Each stack has a diameter of 22 ft. and a top elevation of 258 ft. above the boiler-room floor, with provision for a possible future extension of 50 ft. A wind pressure of 25 lb. per sq. ft. of projected area was assumed on the stack and a maximum compression of 10 000 lb. per sq. in. was used in designing the steel plates. The base of the stack is anchored to the supporting girders by thirty-two bolts 2¼ in. in diameter. The wind shear on the stack is carried down to the foundation through a system of wind bracing between the boiler-house columns.

The turbine room is served by two electric traveling cranes having a span of 96 ft., one with a capacity of 110 tons and the other of 90 tons. Each bridge girder of the heavy crane has two $\frac{7}{16}$-in web-plates, 8 ft. 8 in. deep at the center and 3 ft. 8 in. deep at the trucks. The weight of one bridge girder is 75 000 lb.

The end walls of the turbine room are braced by horizontal wind trusses 6 ft. deep, spanning 98 ft. These trusses are located about mid-way between the basement floor and the bottom chord of the roof trusses, a distance of 108 ft. Between the basement and the wind truss and again between the wind

truss and the bottom chord bracing, three vertical columns are framed which are covered with brick pilasters to stiffen the wall.

The main turbo-generators in the turbine room have a capacity ranging from 35 000 to 50 000 kw. The machines are supported on steel foundations in order to provide sufficient space in the basement for the condensers and auxiliaries. Low unit stresses were used in designing the foundations in order to prevent excessive deflection and to minimize vibration. The box columns as well as the pairs of girders supporting the bed-plate of the machine are filled with concrete. Fig. 14 is a view taken from the basement floor of the turbo-room showing the heavy steel construction supporting the turbo-generators.

An interesting feature in connection with the design of the structural steel of the boiler-house, in addition to supporting the brick-lined steel stacks, is the hanging of the boilers from the structural steel. The water tubes and the steam drums are hung from the girders above and the steel and frame casing enclosing the boilers and stokers is supported on the floor below. Fig. 15 shows the boiler hung in position before the enclosing walls and stokers were erected.

Since the design of the electrical switching apparatus required the separation of the phases it became necessary to give some consideration to the heating caused by induced currents in the structural steel. To minimize this effect it was decided to build this part of the structure in reinforced concrete. In this design the steel is reduced to a minimum and the contact of the various parts of the reinforcement forming loops around the single-phase conductors is too poor to permit any appreciable rise in temperature. The electrical switching apparatus is housed in concrete compartments producing very heavy floor loads and requiring a reinforced concrete construction of rather an exceptional weight.

The first section of the switching galleries which take care of approximately one-half the total capacity of the station is 120 by 105 ft. and seven stories high. The concrete mixing plant used for this part of the work was located on the west side of Locust Avenue. The concrete was shot from a lifting tower, 220 ft. high, to a distributing tower across the street, 200 ft. high. The mixer was of ¾-cu. yd. capacity. The work on this first section of the electrical galleries, involving 6 600 cu. yd. of concrete and 574 tons of reinforcing bars, was started on February 2, and completed on March 29, 1921. Fig. 16 shows the first section of the reinforced concrete switching galleries under construction. This work is independent of the boiler-house, the structural steel framework of which is also shown.

The separation of the phases is effected by continuous walls running through all the floors designed so that it was possible to introduce ventilating shafts communicating with the various floors and arranged so that in addition to the continuous ventilation required it would be possible to remove the smoke quickly from burning oil or insulation should any short circuit to ground take place within any one of the sections.

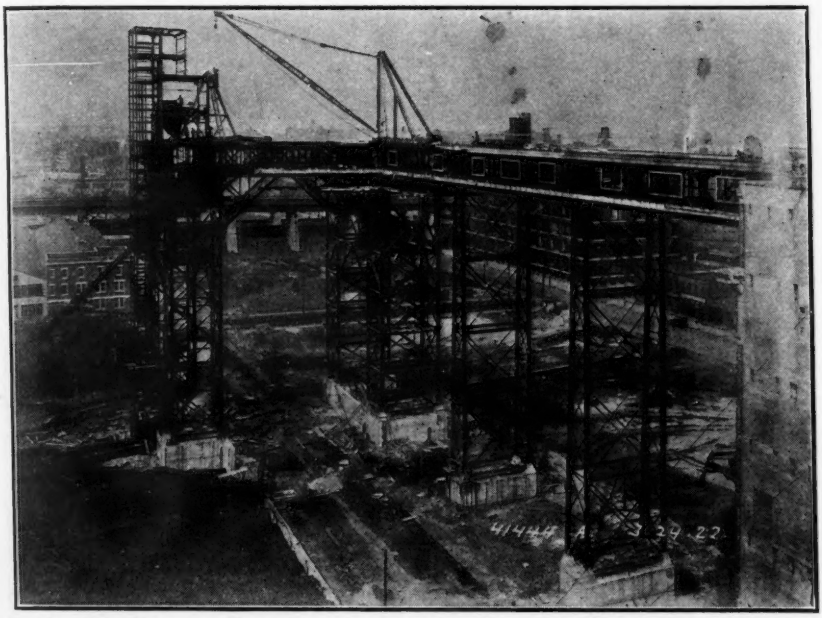


FIG. 13.—VIEW OF STEEL TRESTLE CARRYING TRACKS ACROSS LOCUST AVENUE, HELL GATE STATION.

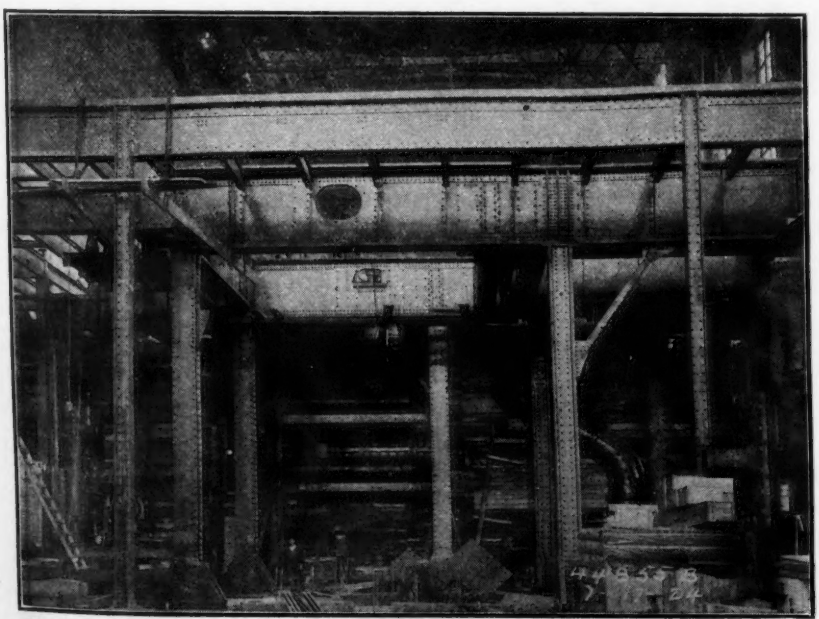


FIG. 14.—VIEW FROM BASEMENT FLOOR OF TURBO-GENERATOR ROOM, SHOWING HEAVY STEEL CONSTRUCTION SUPPORTING TURBO-GENERATORS, HELL GATE STATION.

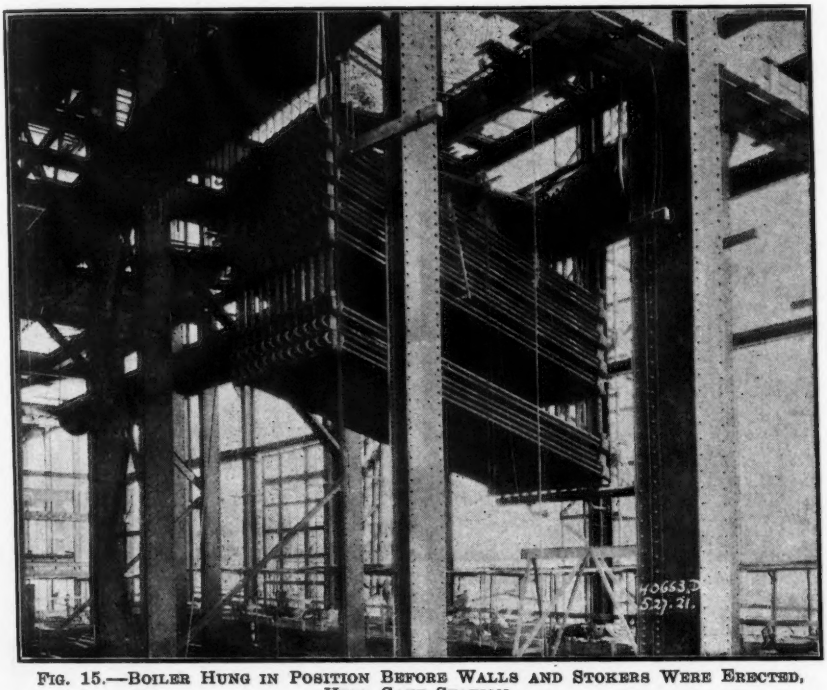


FIG. 15.—BOILER HUNG IN POSITION BEFORE WALLS AND STOKERS WERE ERECTED, HELL GATE STATION.

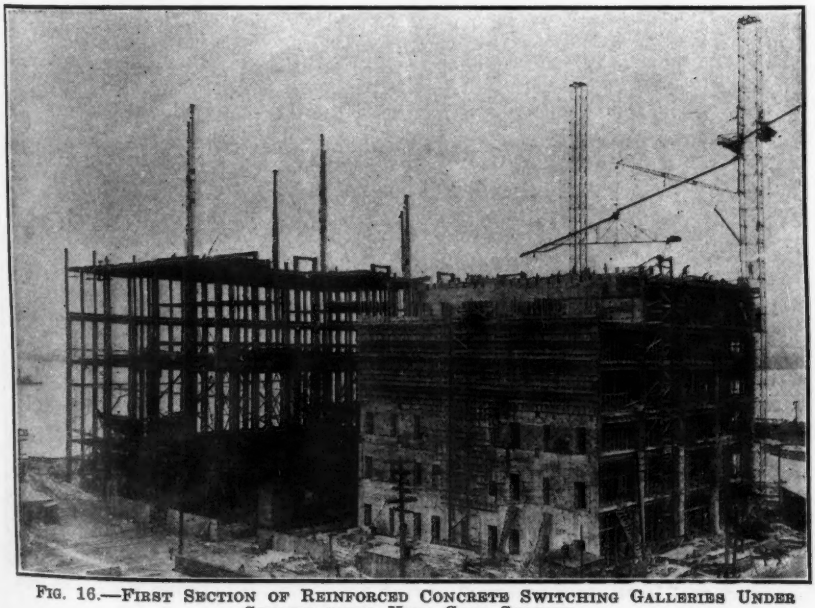


FIG. 16.—FIRST SECTION OF REINFORCED CONCRETE SWITCHING GALLERIES UNDER CONSTRUCTION, HELL GATE STATION.

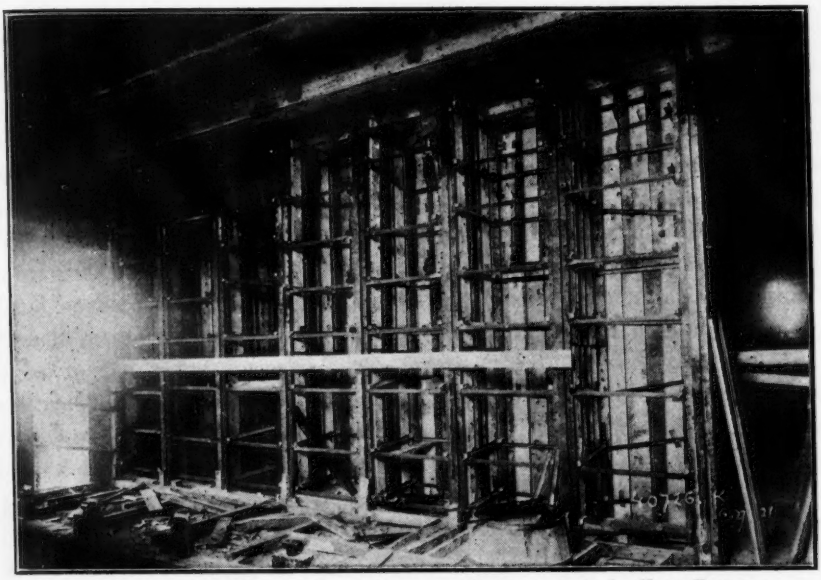


FIG. 17.—FORMS USED IN CONCRETING IN CONSTRUCTION OF HIGH-TENSION SWITCH COMPARTMENTS, HELL GATE STATION.

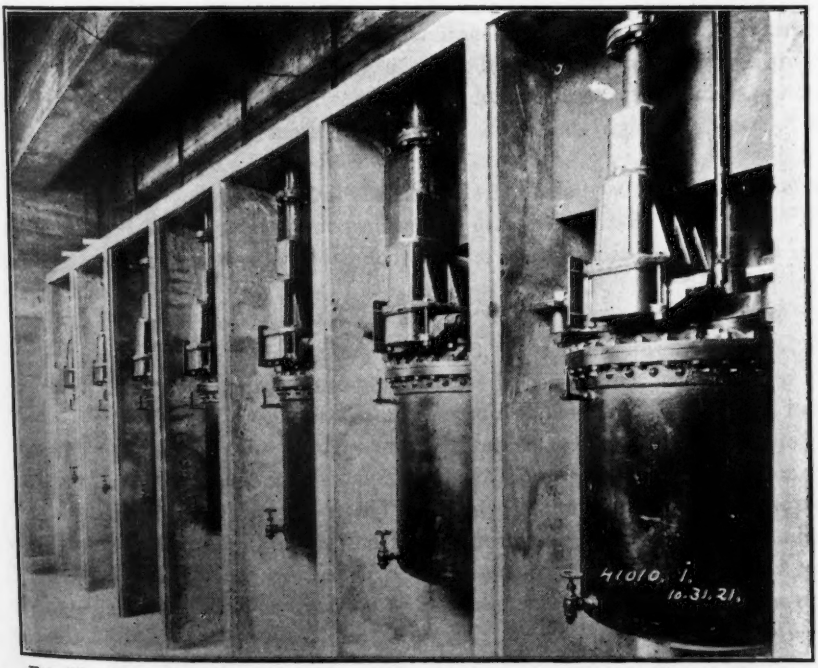


FIG. 18.—VIEW SHOWING HIGH-TENSION SWITCH COMPARTMENTS, HELL GATE STATION.

While the high-tension switch compartments might be considered a part of the electrical switching equipment it is equally true that they are part of the structure of the building and the problems involved in their design and construction belong to the structural engineer's field. The switching equipment is probably the most interesting part of this station and to describe it properly would require a volume in itself. Those who are interested will find many excellent descriptions in the technical press.* In view of the duplication of detail in the construction of the compartments, of which there are about 3 000, a special effort was made to produce a form for concreting which could be used many times and which was readily demountable. Estimates were prepared comparing wooden with metal forms and economy seemed to point to the metal forms. These forms (Fig. 17) were constructed and assembled in the shop of the manufacturer and bolt holes and blocking were introduced where necessary to hold the inserts, which formed a part of the electrical construction, within the concrete of the compartment. The forms proved satisfactory except that in special places the sheet metal used was too light and it became necessary to introduce wooden struts. This, however, was inexpensive and successful.

The actual experience in construction again opened the question of the relative cost of metal *versus* wooden forms and in the second section of the electrical galleries, built about two years after the first ones, wooden forms were used throughout. One of the advantages of wooden forms as compared with metal forms is the flexibility of introducing and changing the inserts which could be very readily attached to wooden forms in any desired position. The inserts, numbering more than 30 000, form an important part of the detail of construction

The compartments in general are about 4½ ft. wide by the full height of the floor, with 4-in. concrete side-walls and 6-in. rear wall (Fig. 18). The side-walls are rabbeted to receive doors which are hung in position and made readily removable.

A detail which proved most satisfactory was designed in connection with the acid-proofing of the storage-battery floors. It was absolutely essential that the sulfuric acid from the storage batteries be prevented from coming into contact with the reinforcing rods and structural steel members of the building, and the double acid-proofing method was introduced. Between the first layer of acid-proofing and the surface layer there was a layer of concrete about 3 in. thick. The effect of the acid on the concrete is to increase appreciably the volume and if the acid penetrates the top layer it would be evident by the swelling of the acid-proof surface, permitting a repair to be made and the defective concrete to be removed before the acid could penetrate the second layer.

* Particularly in the *Electrical World* and the publications of the United Electric Light & Power Co.

The drain from this floor is a lead pipe terminating in a non-corrosive metal drain box. Fig. 19 is a cross-section of the acid-proof floor construction through the drain box.

The quantity of water pumped through the condensers has often been referred to on account of the very large volume required. When the station is completed there will be eight condensers (Fig. 20) ranging in size from 50 000 to 65 000 sq. ft. The quantity of water varies from 70 000 to 90 000 gal. per min. On the assumption that all pumps are operating, the quantity of water pumped through the condensers would be 640 000 gal. per min., or 940 000 000 gal. per day. On a 40% load factor the quantity pumped would be 376 000 000 gal. per day. A better idea of what these figures mean is obtained by comparing this quantity of water with that delivered each day to the entire City of New York, which is about 850 000 000 gal.

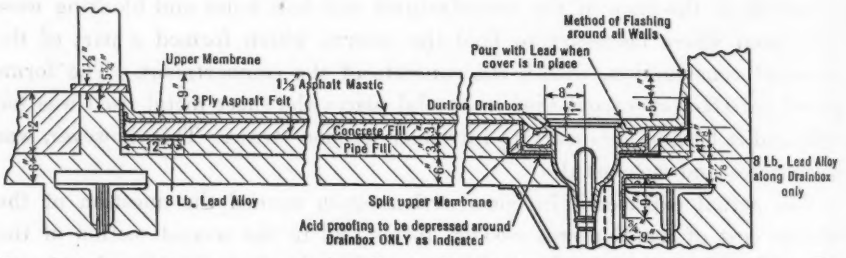

Method of Flashing around all Walls
Pour with Lead when cover is in place
Upper Membrane
1½ Asphalt Mastic
3 Ply Asphalt Felt
Duriron Drainbox
Concrete Fill
Pipe Fill
8 Lb. Lead Alloy
Split upper Membrane
Acid proofing to be depressed around Drainbox ONLY as indicated
8 Lb. Lead Alloy along Drainbox only

FIG. 19.—CROSS-SECTION OF THE ACID-PROOF FLOOR CONSTRUCTION, THROUGH DRAIN BOX, HELL GATE STATION.

The energy for pumping this enormous quantity of water is relatively small, since the only work required is to overcome the friction in the pipes and condensers. The whole system is operated as a siphon and the height to which the water is raised (approximately 30 ft.) does not materially affect the amount of energy required to produce circulation. Fig. 21 shows the two 54-in. suction pipes and the two 35 000-gal., motor-driven centrifugal pumps.

To insure against the possibility of the fresh-water supply (feed water) being cut off from this station arrangements were made with the city to receive water from three independent sources. The first taps the water supply at 138th Street and Southern Boulevard; the second, on a different main, from 133d Street and Southern Boulevard; and the third from the local water main immediately adjacent to the station.

The proximity of the New York, New Haven and Hartford Yard, immediately south of 132d Street, made it possible to install at this station a railroad siding (Fig. 12) which is frequently used for receiving equipment. The possibility of receiving coal on this siding was considered but it is not used to any extent since the cheapest coal that could be received in the harbor naturally comes by boat. As an emergency measure, however, coal might be received by rail.

The exterior finish of the building is a red face brick with precast imitation granite base and trim. The reinforced concrete electrical galleries are veneered with the same brick secured to the concrete by metal clips which

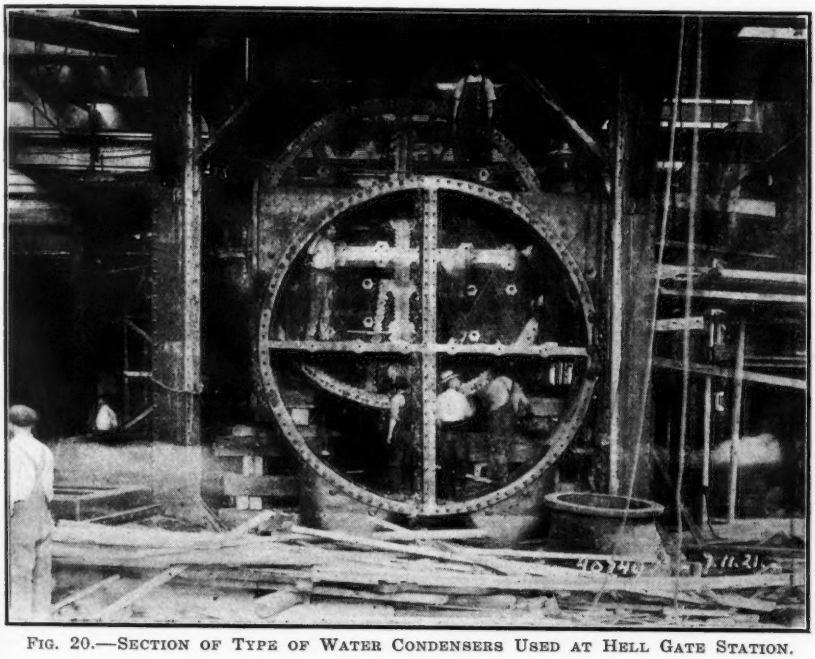


FIG. 20.—SECTION OF TYPE OF WATER CONDENSERS USED AT HELL GATE STATION.

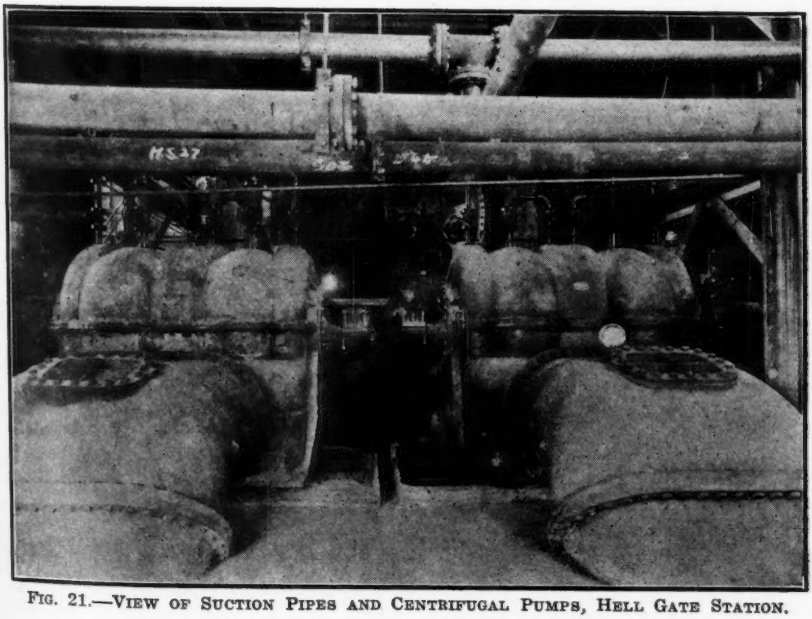


FIG. 21.—VIEW OF SUCTION PIPES AND CENTRIFUGAL PUMPS, HELL GATE STATION.

construction gives an outward appearance of continuous steel and brick construction. The exterior window frames and sashes are of metal, the size of the openings being designed to use standard frames of a stock pattern.

The Hell Gate Station was put into service in the fall of 1921. At present (1927) there are seven turbo-generator units in operation with an aggregate capacity of 275 000 kw., of which 105 000 kw. has a frequency of 25 cycles and 170 000 kw. a frequency of 60 cycles. There is also a frequency changer of 35 000 kw. capacity. The eighth and last generator of 160 000 kw. capacity has been ordered and will be in service in 1928.

THE CIVIL ENGINEERING FEATURES OF THE KEARNY POWER STATION*

By Roman von Fabrice,† Assoc. M. Am. Soc. C. E.

Synopsis

The object of this paper is to set forth the problems that the civil engineer is called upon to solve in order to make "super-power" a possibility. To this end the civil engineering features of the design and construction of the Kearny Power Station are treated, each phase of the work separately.

The Kearny Power Station is situated on the west bank of the Hackensack River in the Town of Kearny, N. J., about midway between Newark and Jersey City, and is interconnected with the Essex and Marion Stations by a 132 000-volt transmission line supported on steel towers. It is the largest electric generating station in the State of New Jersey and will have a capacity of more than 400 000 kv-a. when completed.

"Super-power" means the production of electrical energy in great magnitudes and its distribution over long distances. Among the public, as well as among the Engineering Profession in general, the impression has been created that developments for this purpose primarily concern electrical engineers. This is erroneous, and although it is true that electrial engineers are responsible for the electrical features, the mechanical engineers and civil engineers are entitled to equal recognition in the problem of making super-power a possibility.

This paper will show the importance of the problems which the civil engineer was called upon to solve in this station, such as foundations and superstructures, high-tension sub-stations, transmission lines, coal and ash-handling structures, cooling water systems, etc. To provide, in co-operation with the architect, the proper structures for the accommodation of the equipment selected by the electrical and mechanical engineers, constitutes the chief problem of the civil engineer in such undertakings.

In order to accomplish these results properly he must be familiar, not only with the construction and requirements of the various types of electrical and mechanical equipment, but also with their operation, so as to be able to produce suitable and economical structures both for the support and housing of such apparatus.

General Description of Plant and Property

The Kearny Power Station, New Jersey's largest and newest generating station, built by the Public Service Production Company for the Public

* Presented at the meeting of the Power Division, New York, N. Y., January 21, 1926.

† Designing Engr., Public Service Production Co., Newark, N. J.

Service Electric Power Company, is situated on the west bank of the Hackensack River in the Town of Kearny, N. J. (Fig. 1). The site selected is near the load center, has adequate condensing water facilities, and offers both rail and water transportation for the fuel required for the continuous and commercial operation of the station. The property on which the station is located comprises 78.9 acres of salt-water meadow land, partly submerged at high tide and requiring piling for the support of the structures.

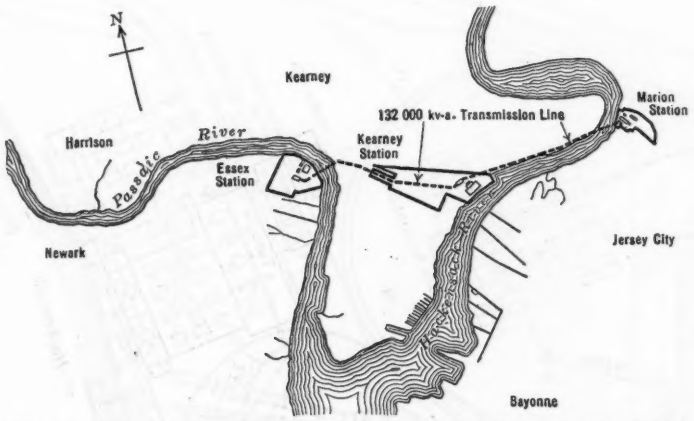


FIG. 1.—MAP SHOWING RELATIVE LOCATIONS OF KEARNY, ESSEX, AND MARION POWER STATIONS.

The initial capacity of the station is 205 100 kv-a., this being the largest ever installed at one time. The ultimate capacity, after the completion of the second half, will be more than 400 000 kv-a.

Beginning at the river the various buildings and structures are arranged in the following sequence (Figs. 2 and 3): Coal-bunker, boiler-house, turbine room, switch-house, service building, high-voltage outdoor sub-station, and transmission towers, the coal and ash-handling system and coal storage being to the west of the other structures. This arrangement was determined after most careful consideration as to the cost of fuel delivery, utilization of real estate, railroad sidings, fuel-handling and storage, cooling water supply, electrical distribution, and economy of design.

ARCHITECTURAL FEATURES

All buildings have been designed to act as parts of a single unit, the central axis of the plant passing through the center of each building. The service building and switch-house form the central motif of the whole group. The main elevation of the turbine room consists of a series of typical bays, the end bays being paneled. The buildings are classical in style, the Corinthian order being utilized where enrichment occurs. As far as was consistent with power-house architecture, projections were held flat and details were refined. A striking feature is a cast-iron Palladian window, 70 ft. high, at

the end of the turbine room and a similar but somewhat smaller one in the center of the front of the switch-house (both are seen in Fig. 4).

The interior of all buildings is severely utilitarian with the exception of the turbine room (Fig. 5), which is finished with gray face-brick panels and pilasters and a red tile floor. The control room in the switch-house has plastered walls and ceiling, a plaster cornice, and a rubber tile flooring.

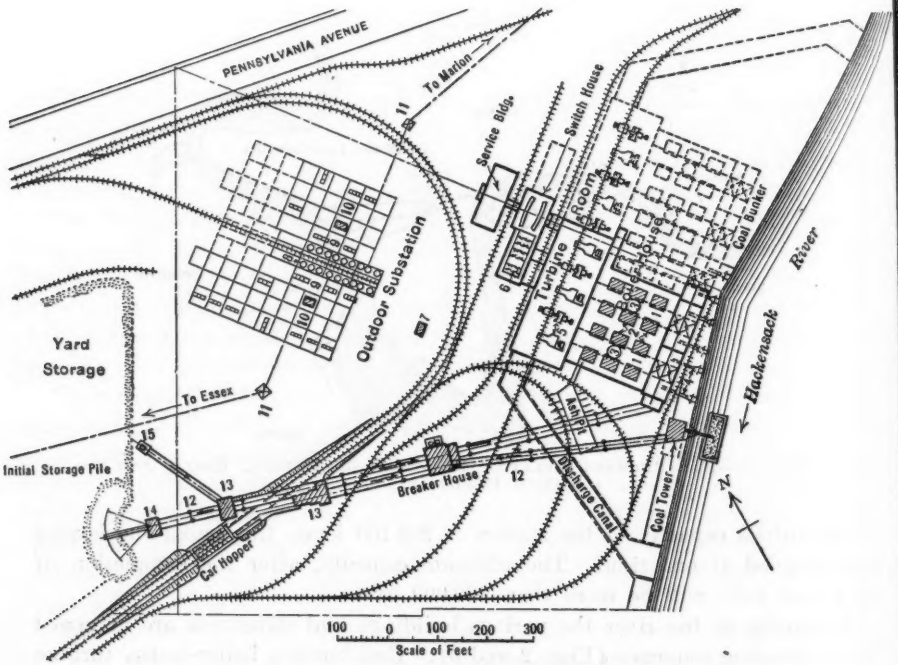


FIG. 2.—PLAN OF KEARNY POWER STATION.

The exterior walls of all buildings are of common red brick trimmed with limestone. The general architectural appearance of the entire station is dignified, quiet, and massive in character.

GENERAL SPECIFICATIONS

The structural steel for Kearny Power Station was designed on the basis of a stress of 18 000 lb. per sq. in., as against the usual practice of 16 000 lb. per sq. in. This economy effected a saving of approximately 12% in the weight of steel. Wind-bracing was designed for a wind load of 30 lb. per sq. ft. and a maximum allowable fiber stress of 24 000 lb. per sq. in. Wherever possible tension members were used; in addition, many other features of economy were introduced, such as three-hinged trusses, the maximum application of cantilever and suspended sections, and the adoption of rolled shapes rather than lattice sections.

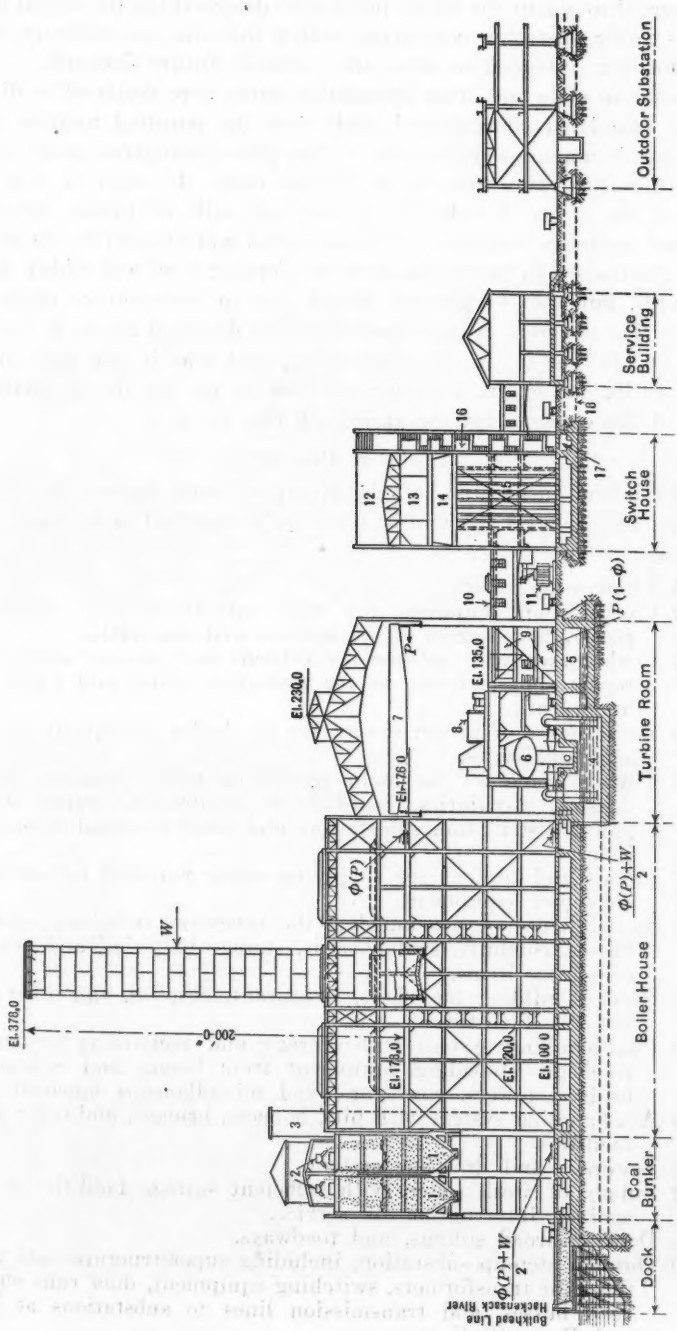


FIG. 3.—CROSS-SECTION THROUGH KEARNY STATION.

All floors throughout the entire plant were designed for the actual loadings obtainable under operating conditions, rather than for an arbitrary assumed uniform loading, intended to meet any probable future demands.

The concrete mats and other foundation parts were designed to distribute safely the calculated concentrated loads over the required number of piles with the minimum mass of concrete. The piles throughout were computed for a maximum carrying capacity of 15 tons each. In order to increase the stiffness of the entire foundation all vertical walls of intake tunnels and suction and discharge chambers were considered and utilized as rib reinforcement for the mats. In many instances reinforcing steel was added, not only for strength, but also to prevent cracks due to temperature changes and shrinkage. The concrete in the foundations was designed for 40 lb. per sq. in. in shear, 600 lb. per sq. in. in compression, and that in the floor and roof slabs for 40 lb. per sq. in. in shear and 700 lb. per sq. in. in compression. The mix of the concrete for the entire job was 1:2:4.

Scope of Project

For the initial installation of the Kearny Power Station the following major items are included and will be more fully described under their respective headings:

(1) Preparation of site.
(2) Construction buildings for field superintendence, maintenance, repair, and storage of equipment and materials.
(3) Coal-bunker with adequate overhead coal storage capacity, and repair shops, storeroom for materials, toilet and locker rooms underneath.
(4) Boiler-house of proper design for the boiler equipment and necessary auxiliaries.
(5) Turbine room for the main generating units, complete with condensers, auxiliaries, heat-balance equipment, cranes, auxiliary power distribution sub-station, and other essential miscellaneous items.
(6) Canals and tunnels for supplying water required for service and condenser equipment.
(7) Switch-house to accommodate the necessary switching equipment, busses, conduits, switchboards, storage batteries, and other miscellaneous items.
(8) Service building for offices, restaurant, hospital, and other administrative facilities.
(9) Coal-handling system with storage and reclaiming system, yard trackage, unloading equipment from barges and railroad cars, bridges, towers, conveyors, and miscellaneous apparatus.
(10) Ash-handling system with pits, hoppers, bridges, and other disposal facilities.
(11) Sewerage and drainage system.
(12) Supply of fresh water, with sufficient storage facilities to insure against interruption of service.
(13) Dock, railroad sidings, and roadways.
(14) Outdoor step-up substation, including superstructures and foundations for transformers, switching equipment, duct runs and overhead busses, and transmission lines to substations at Marion and Essex Stations.
(15) Transformer stations at Essex and Marion Stations.

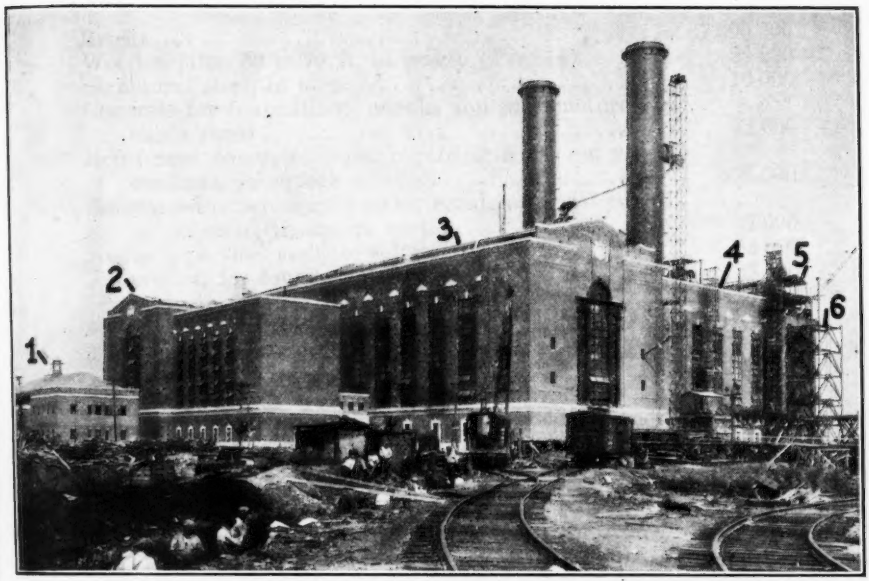


FIG. 4.—GENERAL VIEW, KEARNY POWER STATION: (1) SERVICE BUILDING; (2) SWITCH HOUSE; (3) TURBINE ROOM; (4) BOILER-HOUSE; (5) COAL BUNKER; AND (6) HEAD TOWER OF CONVEYOR TRESTLE.

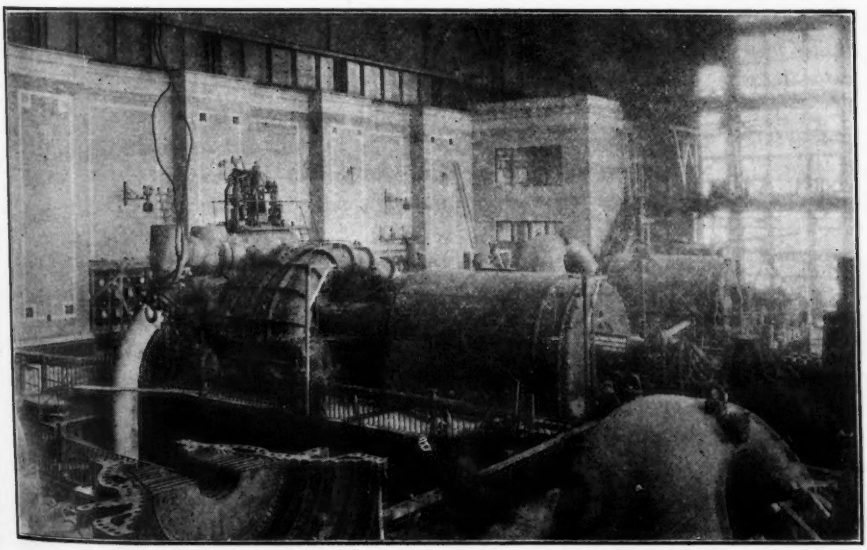


FIG. 5.—INTERIOR VIEW OF TURBINE ROOM, KEARNY POWER STATION.

The materials required for the initial construction at the Kearny Power Station are as follows:

Bricks	7 500 000
Wooden piles, 30 to 70 ft. in length (Fig. 14)	29 000
Structural steel, in tons	16 000
Concrete for foundations, canals, and substructures, in cubic yards	72 000
Reinforced concrete floors, requiring 5 000 cu. yd. of concrete, in square feet	235 000
Reinforced concrete roof slabs, requiring 1 900 cu. yd. of concrete, in square feet	92 000
Heavy type steel sash, in square feet	35 500
Excavation for foundations and canals, in cubic yards	125 000
Dredging in front of the dock, in cubic yards	120 000
Iron conduit, in miles	68
Fiber conduit, in miles	10
Electrical wire (in cables), in miles	585
Boiler tubes, 4 in. in outside diameter, in miles	46
Admiralty condenser tubes, 1 in. in outside diameter by 16 gauge, in miles	184
Lumber for dock, in board-feet	1 300 000
Cinders for fill, in cubic yards	700 000

These items do not include the lumber used for forms, steel required for reinforcement for canals, foundations, floor and roof slabs, and steel for flues, ducts, and galleries, nor brickwork for boiler settings. The volume of all buildings, exclusive of those for the coal-handling system, is 10 500 000 cu. ft.

Preparation of Site

Careful consideration had to be given, when laying out the construction program, to the proper location of the various final structures and of the temporary construction sheds, office buildings, railroad tracks, storage space, etc. (Fig. 6), in order not to interfere with the permanent buildings. The swampy nature of the land (Fig. 7) made it necessary to build embankments for railroad tracks and to improve and extend the present roads across the marshes linking them to the existing main highways for access of heavy trucks to the site (Fig. 7).

About 700 000 cu. yd. of ashes hauled to the site by motor trucks were used in filling the land on which the various construction buildings (Fig. 8), roadways, and railroad tracks were located and to provide space for storage of materials. Office buildings for the field force, wash and locker rooms for the workmen, a hospital, storage houses for material and smaller apparatus, a compressor plant, a distribution substation, and various work and repair shops were also erected to prepare for actual construction of the project.

The tracks (Fig. 6) were laid in positions to give easy access for the storage of the various materials and to handle them effectively when required during construction, and not to interfere with permanent structure.

A fresh-water supply for operating the final station as well as for construction purposes had to be laid across the meadows from the city mains. Similar provisions were made for sanitation.

An unloading station for sand, rock, and gravel was erected at the river and connected by a trestle to the concrete mixing plant and distribution tower, the position of which had been selected so as to give uninterrupted service during the construction of the first half of the ultimate station.

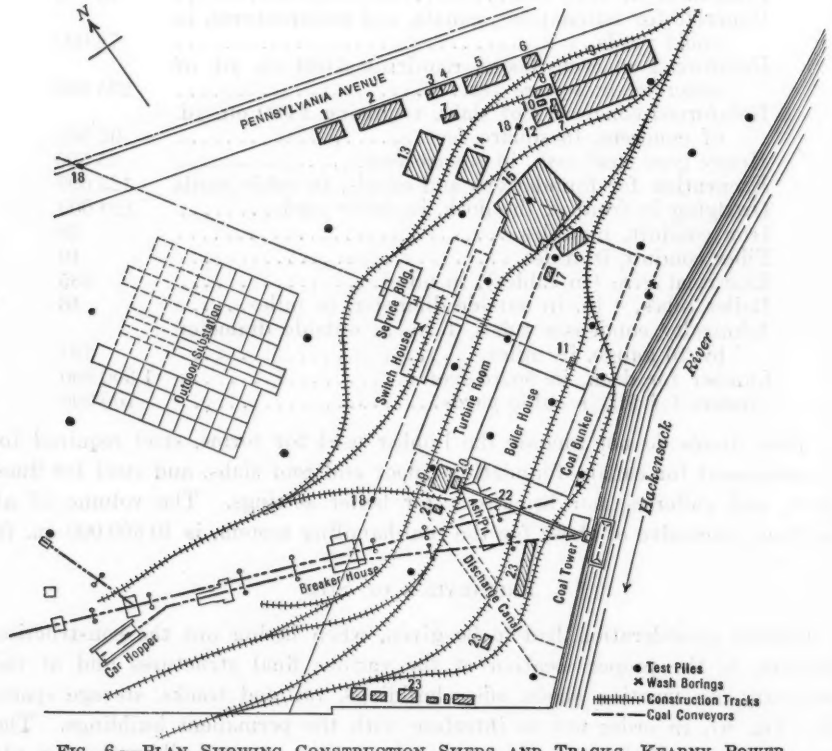


FIG. 6.—PLAN SHOWING CONSTRUCTION SHEDS AND TRACKS, KEARNY POWER STATION.

A series of test piles (Fig. 6) were driven to determine the length and carrying capacity of the piling; wash and core borings were also made to ascertain the nature of the subsoil. These borings showed that the strata under the station site were composed of a layer several feet in thickness of vegetable matter underlain by a mixture of clay and sand with vegetable matter, then layers of sand, becoming coarser as the depth increased, and, finally, shale and rock. In some places strata of quicksand were also found.

In order to keep the site from being flooded by the tides a temporary timber bulkhead (Fig. 9) was constructed along the water-front, and earthen dikes along the sides of the excavation. The excavation was kept dry by pumping.

COAL-BUNKER

The coal-bunker (Figs. 2, 3, and 10) is on the water-front of the station. In order to insure the continuous operation of the plant, an ultimate bunker capacity of 19 000 tons was required. The grouping of the boilers around

FIG. 7.—BUILDING SITE SHOWING MARION POWER STATION IN BACKGROUND: (1) MARION STATION; (2) WEST END GAS PLANT; (3) SEABOARD BY-PRODUCTS COMPANY; (4) HACKENSACK RIVER; AND (5) PILE-DRIVER DRIVING TEST PILES.

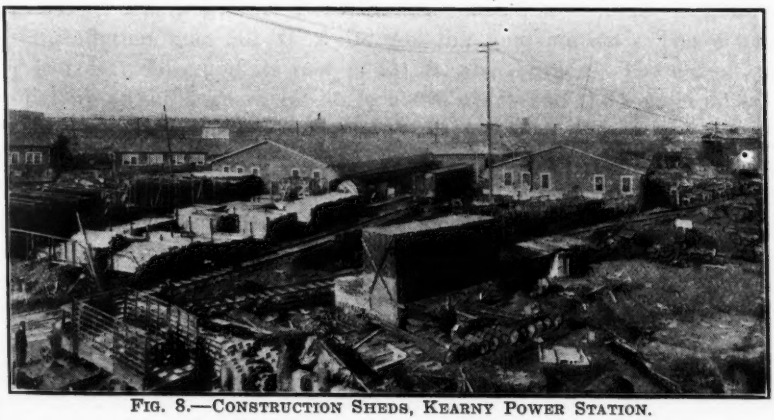


FIG. 8.—CONSTRUCTION SHEDS, KEARNY POWER STATION.

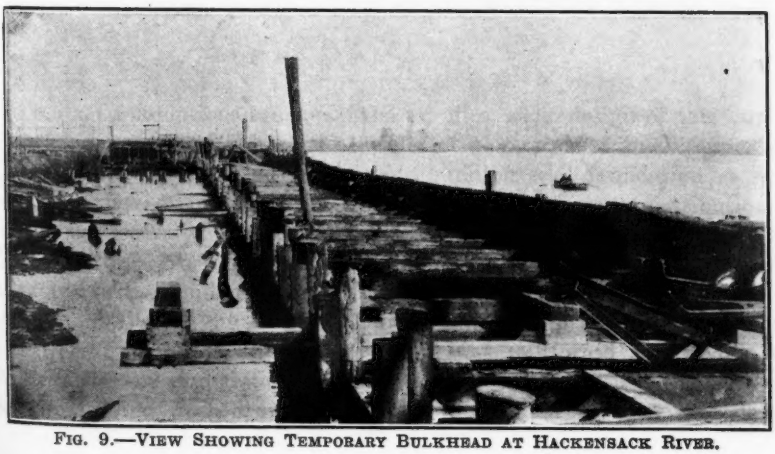


FIG. 9.—VIEW SHOWING TEMPORARY BULKHEAD AT HACKENSACK RIVER.

five stacks and firing aisles also necessitated the planning of five individual bunkers, each longitudinally divided by a center wall into two pockets. The initial installment has two bunkers of a total capacity of 7 600 tons, serving twelve boilers.

The coal is conveyed from the bunkers to the boilers by a traveling weigh-larry having a capacity of 25 tons. The bottom elevation of the bunkers was determined by the height of the stoker hoppers, and the head-room needed for the coal larry. The resulting space below the bottom of the coal-bunker was utilized to house the machine and repair shops, stores for material for station equipment maintenance, wash and locker rooms, toilets, and other minor facilities (Fig. 10). The bunker building is of steel frame construction with concrete slabs for floors and coal-pockets. The vertical bunker walls are of concrete, while the walls for the store-rooms are of brick. The longitudinal center wall, dividing each bunker into two pockets, is of hollow construction and designed to withstand the horizontal thrust due to the coal with either pocket full. The bunker walls have smooth surfaces to prevent the coal from arching and are tied together by I-beams protected by concrete (Fig. 10).

The bunker building is 43 ft. 9 in. wide, and 236 ft. 5 in. long for the initial installation and 501 ft. 6 in. for the final station. The conveyor gallery is 114 ft. long and its roof is 131 ft. above grade. The cut-off gates at the bottom of the hoppers are 51 ft. above grade and the maximum height of coal stored in the bunkers is 60 ft.

The inclined bottom of the coal-bunkers forms the roof of the store-room buildings below. The heavy vertical loads in the bottom of the pockets were taken by steel plate girders, while the hopper shape itself was obtained by the use of concrete, reinforced to take the horizontal components of the loads (Fig. 11). This proved more economical than a suspended hip-and-valley steel construction which would have been necessary to form this lower portion.

All calculations for the coal-bunker were based on a weight, w, of 50 lb. per cu. ft. of coal. The horizontal pressure, P, for a depth, h, of coal, and an angle of repose, ϕ, was defined by the well-known formula,

$$P = \frac{1}{2} w h^2 \frac{1 - \sin \phi}{1 + \sin \phi}$$

The bunker foundations are supported on piles. The design of the foundations took advantage of the vertical walls of the intake tunnels under the foundation mat, also utilizing the outside building wall foundation as reinforcing ribs for the comparatively thin mat. A satisfactory distribution of the heavy concentrated column loads was thus obtained. The foundation problem was further complicated by the large wells for the revolving screens installed inside the bunker-house in the intake canals (Fig. 13).

In front of the coal-bunker house was a cut-off wall extending below the line of deepest dredging; this concrete wall was reinforced and tied to the intake tunnels and foundation mat. Its purpose was, first, to prevent the subsoil from being washed out from under the foundation due to dredging;

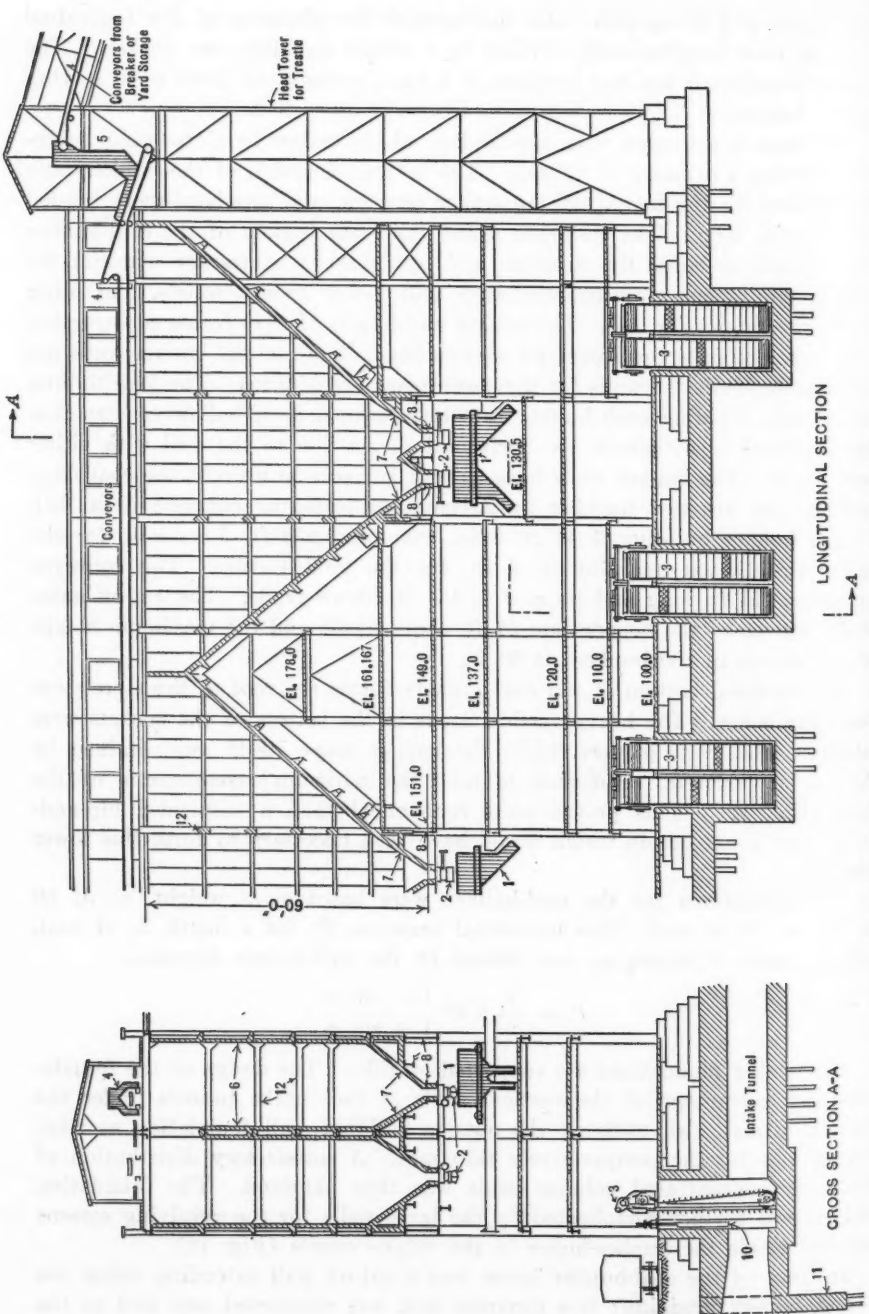


FIG. 10.—SECTIONS OF COAL BUNKER, KEARNY POWER STATION.

and, second, to form a seal against the possible entrance of marine borers in case they should infest these waters at some later date.

The bunker building is held laterally by the main bracing system of the boiler-house; no other special provisions, except tying to the boiler-house steel, were deemed necessary (see Figs. 3 and 29).

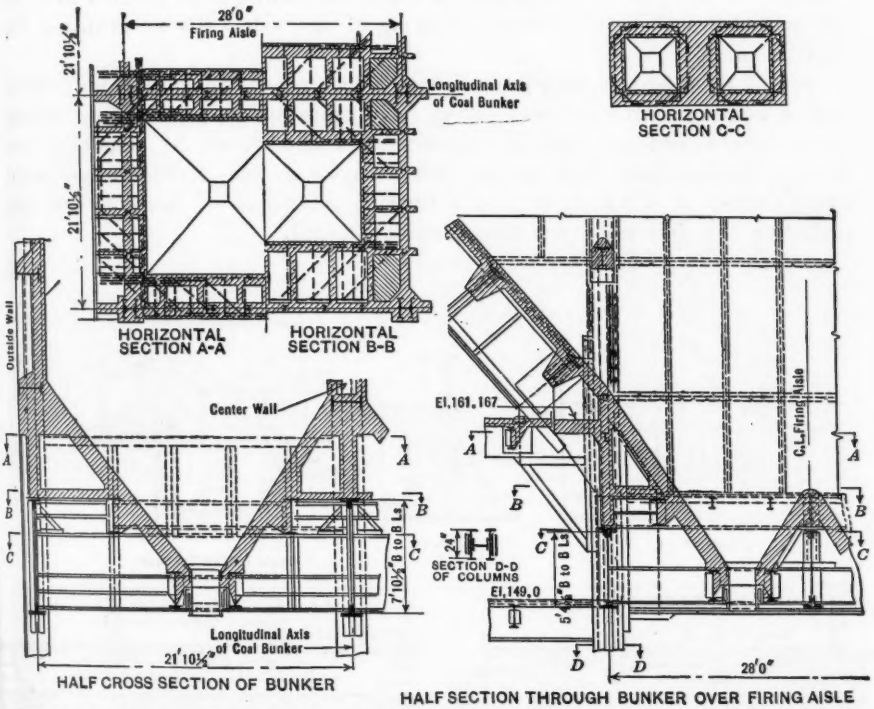


FIG. 11.—DETAILS OF STEEL AND CONCRETE IN BOTTOM OF COAL BUNKERS, KEARNY STATION.

The column loads varied from 475 to 1 450 tons, due to coal, floor loading, and dead loads. The columns are H-beams reinforced with plates or I-beams on both flanges as required by the load. The grillage system (Figs. 12 and 13) consists of two or three tiers of I-beams embedded in concrete. The column bases are steel billets of proper dimensions.

BOILER-HOUSE

The design of the boiler-house steel and foundations (Fig. 14) was governed primarily by the equipment to be installed. Briefly, this equipment consists of twelve Babcock and Wilcox Company 3-pass, water-tube, cross-drum boilers, each having 23 600 sq. ft. of heating surface and 4 130 sq. ft. of superheating surface, designed for a maximum steam pressure in the tubes of 385 lb. (gauge) and 720° Fahr. at 200% of rating and capable of operating at a maximum rating of 350 per cent.

The general layout of the boiler-house is shown in Figs. 15 and 16 and the steel work in Fig. 17. The boiler settings are 22 ft. high and arranged in

FIG. 10.—SECTIONS OF COAL BUNKER, KEARNY POWER STATION.

single units with solid walls and steel jackets. Provisions are made for ventilating and cooling the settings. The brickwork is clear of the building columns and supported on heavy girders designed for the loading imposed and the special requirements for stokers and their mechanism.

The side-walls above the combustion chambers are supported on specially constructed box-girders which form a main ventilating duct from which the air is distributed to the various sections of the side-walls as required for ventilation.

Six of these boilers are equipped with fifteen-retort, Sanford Riley stokers having a projected grate area of 479 sq. ft. of grate surface and furnace volume above the coal bed and including the first pass of 8 985 cu. ft. The other six boilers are equipped with sixteen-retort, Taylor stokers having a projected grate surface of 459.9 sq. ft. and a furnace volume above the coal bed and including the first pass of 8 472 cu. ft. (Fig. 18).

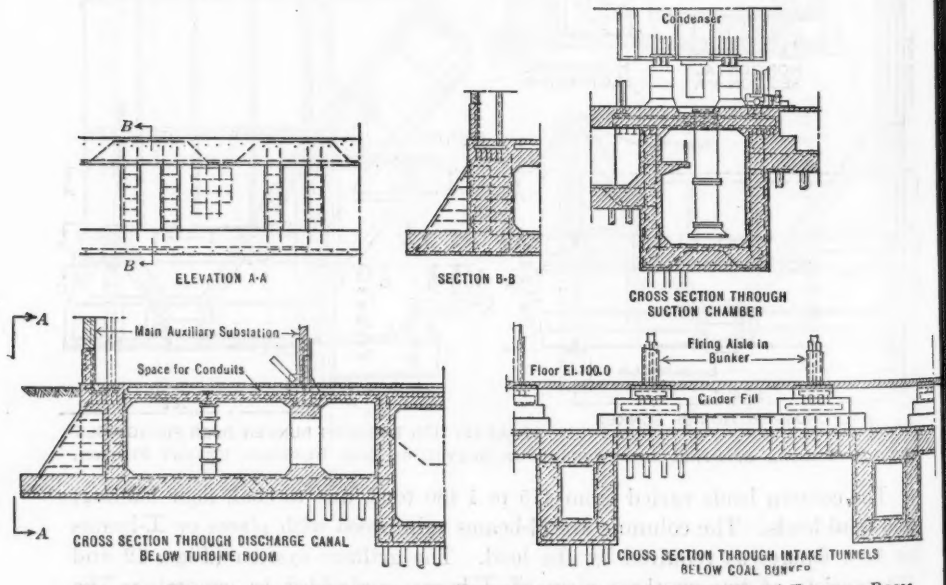


FIG. 12.—DETAILS OF FOUNDATIONS AND REINFORCING IN COAL BUNKER AND TURBINE ROOM.

Each group of three boilers is equipped with a common forced-draft air duct into which three Buffalo Forge Company fans discharge. Each of these fans has a capacity of 134 000 cu. ft. per min. at 65° Fahr. and 7-in. water pressure. The fans are driven by a two-speed, 2 300-volt, 225-h. p. General Electric motor. Two of these fans will supply all the air necessary for combustion under normal operating conditions for three boilers, leaving one fan as a spare unit.

The forced-draft air duct is constructed of brick and concrete, utilizing the main floor as the top and the wall between the ash cellar and the forced-draft fan room as one side. This arrangement (Fig. 16) proved to be more economical in design and operation than the conventional steel ducts.

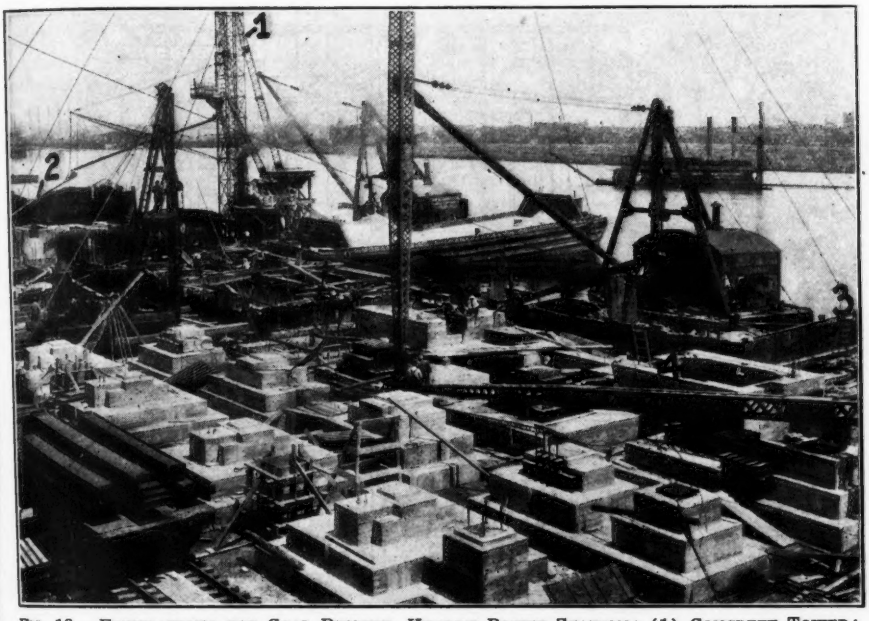


FIG. 13.—FOUNDATIONS FOR COAL BUNKER, KEARNY POWER STATION: (1) CONCRETE TOWER; (2) BULKHEAD; (3) INTAKE; AND (4) SCREEN WELL.

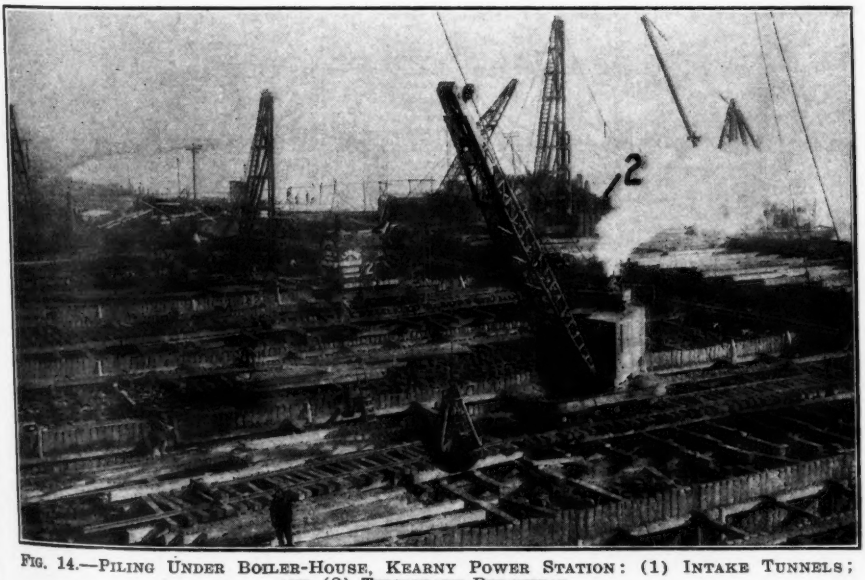


FIG. 14.—PILING UNDER BOILER-HOUSE, KEARNY POWER STATION: (1) INTAKE TUNNELS; AND (2) TEMPORARY BULKHEAD.

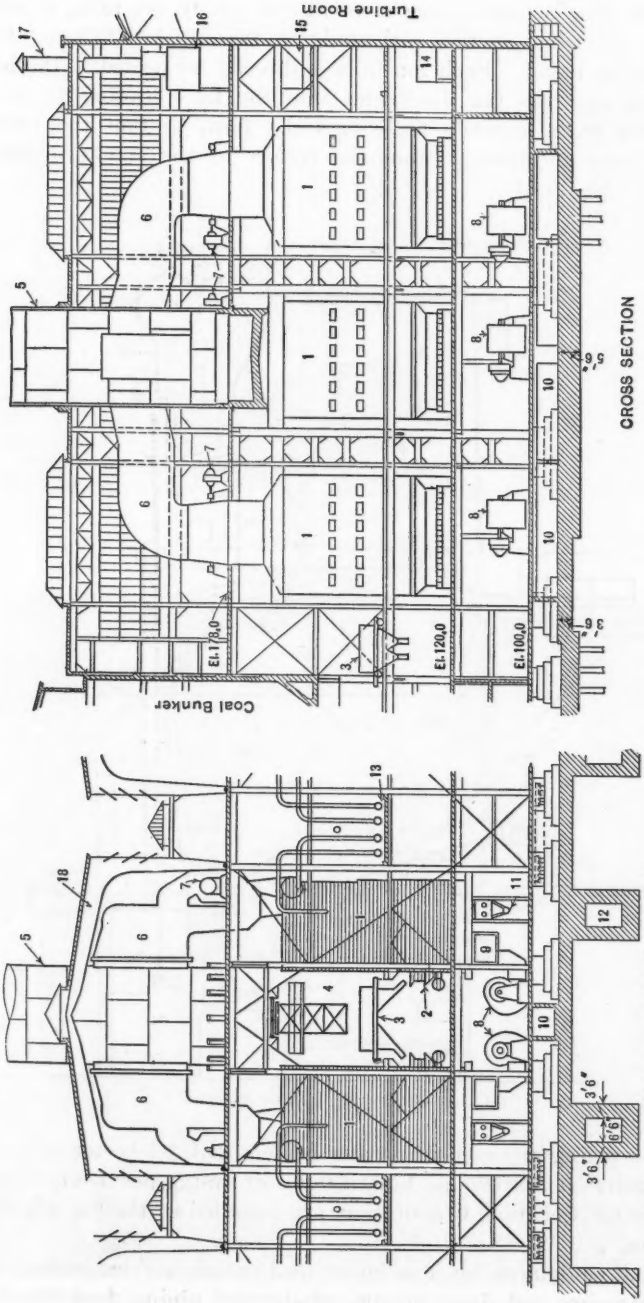


FIG. 15.—CROSS-SECTION OF BOILER HOUSE, KEARNY POWER STATION.

Each boiler has one double width, double inlet, multi-vane Sturtevant induced draft fan, having a capacity of 275 000 cu. ft. per min., at 650° Fahr. and 3.5 in. of water suction, driven by a two-speed, 300-h. p., 2 300-volt General Electric motor. Each fan inlet is directly connected to the breeching of the boiler and the fan discharges into the flues leading to the stacks. Arrangements are also made to by-pass the fans, so that the products of combustion may go directly from the boilers to the stacks under normal rating. (See Fig. 15.)

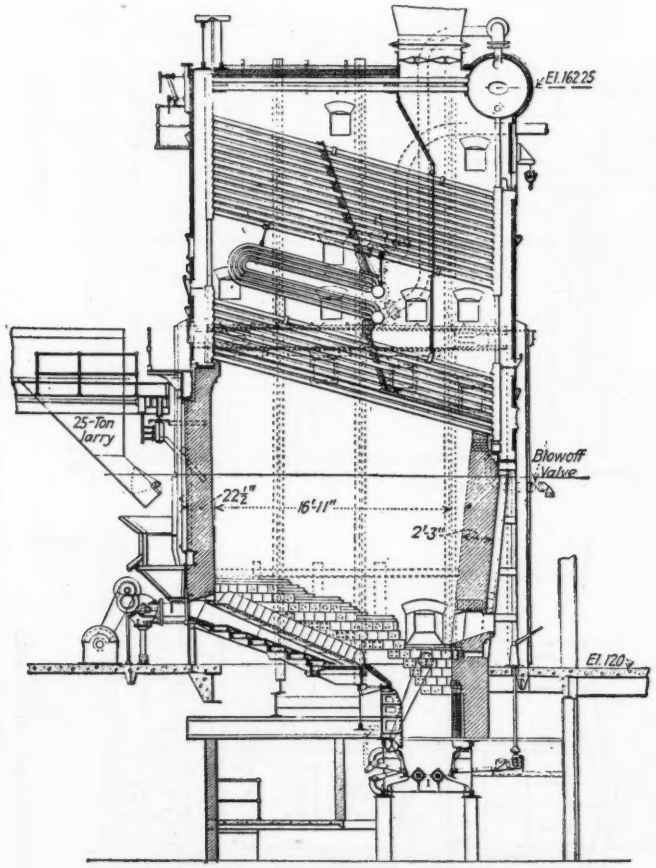


FIG. 16.—PLAN OF BOILER SETTING.

Five de-aerating and metering water heaters with jet heater open feed and with a capacity of 400 000 to 1 000 000 lb. of water per hour, which were furnished by the Cochrane Corporation, are installed in the bay adjoining the turbine room.

Additional equipment, such as boiler feed pumps, service pumps, air compressors, sluiceways and sluice pumps, ash-disposal piping, feed-water heaters, fresh-water storage tanks, service tanks, storage battery, elevators, oil-purifica-

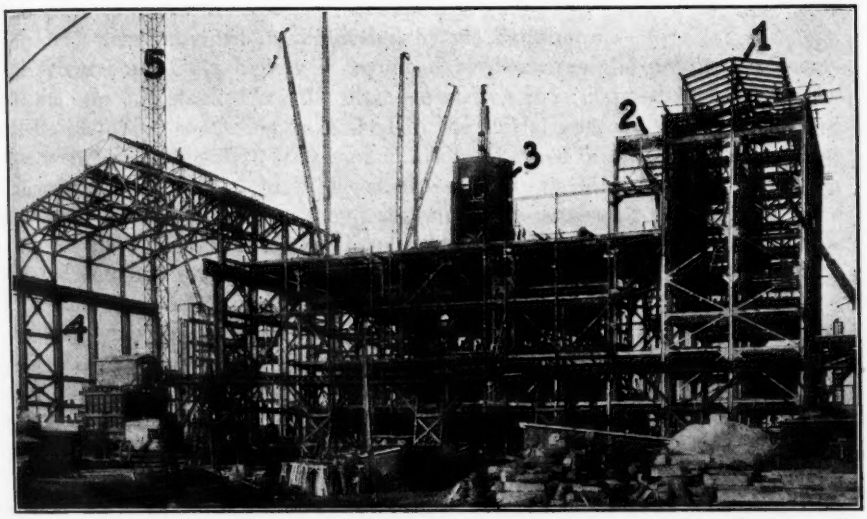
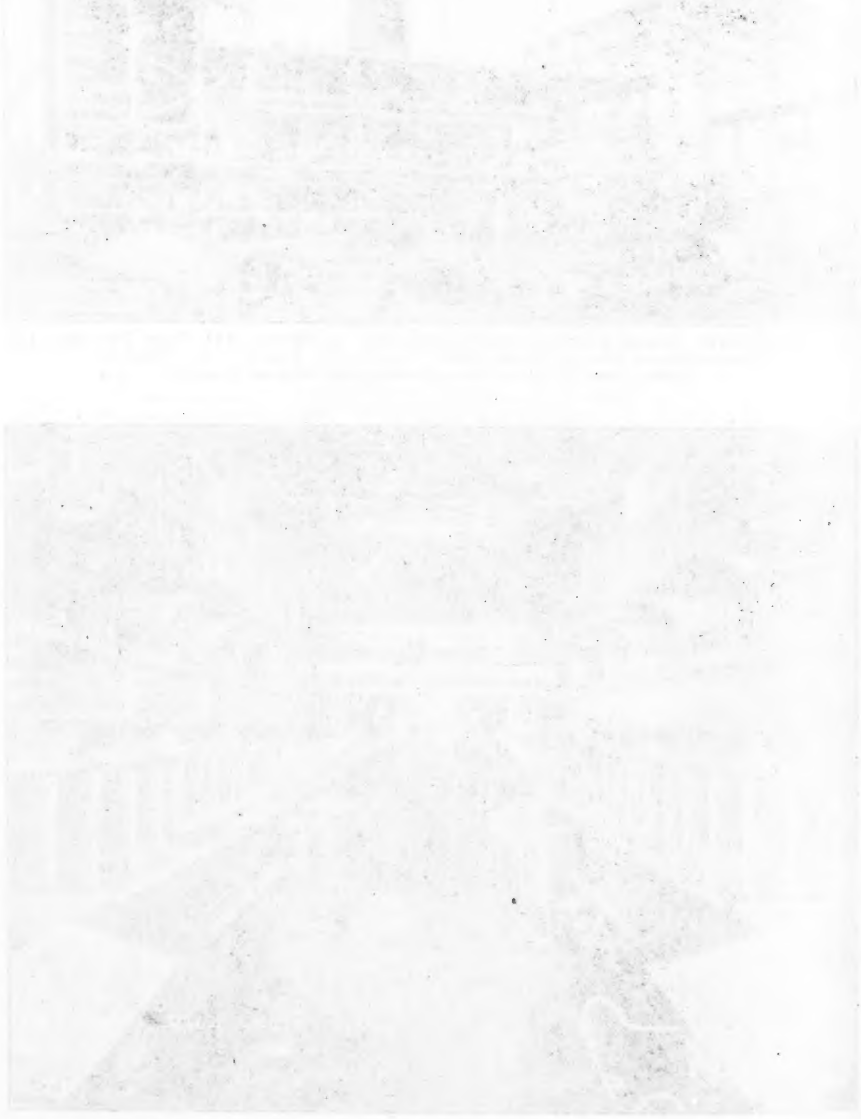


FIG. 17.—KEARNY POWER STATION DURING ERECTION OF STEEL: (1) COAL BUNKER; (2) THREE-HINGED BOILER-HOUSE ROOF TRUSS; (3) STACK; (4) TURBINE ROOM; AND (5) CONCRETE TOWER AND MIXING PLANT.

FIG. 18.—FIRING AISLE AND STOKERS. WEIGH LARRY IN BACKGROUND: (1) STOKERS; AND (2) EXTENSION HOPPERS.



tion system, power-distribution substations, and other incidental equipment (Fig. 15) were provided in the design of the building.

Each group of six boilers is arranged to discharge the products of combustion into one stack. For the ultimate station it is planned that five stacks will be installed, two being furnished in the initial section. The stacks have an exterior diameter of 26 ft. 0 in. and are brick-lined throughout, the internal diameter being 24 ft. 7 in. The stacks are 200 ft. high, and supported on steelwork 78 ft. above grade; they are designed to withstand the wind pressure of a gale blowing 120 miles per hour. This wind velocity striking the cylindrical surface of the stack produces a horizontal wind load equivalent to 33.3 lb. acting upon a vertical plane area. The stacks are built of overlapping steel plates having a thickness of $\frac{3}{8}$ in. at the top and $\frac{3}{4}$ in. at the bottom. These plates are stiffened by vertical angle stiffeners and by horizontal rings made of angles and plates. The horizontal stiffeners are closer together at the bottom than at the top and serve to support and keep the brick lining in position.

The lining was grouted to the steel shell and a space was left between the top of the brickwork and the underside of the next ring or support for the following section. The vertical plate of the horizontal stiffeners was bolted to the angle ring after the lower brick lining was in place and before the following section of brickwork was built.

The stacks are supported on a heavy steel plate-girder platform at the level of the blower floor; the simple arrangement of this platform is shown in Fig. 19. Oblique girders have been avoided as much as possible to simplify fabrication and erection. The anchorage of the steel stack to the supporting structure is also shown. A graphical solution for the determination of the zero line (neutral axis) of the horizontal cross-section at the anchorage is shown on Fig. 19. Since the directions of the wind causing the most unfavorable stresses in the various girders could not be determined offhand, the bending moments, etc., have been found for several conditions, varying with the direction of the wind. The envelope line enclosing these various moment curves (Fig. 19) was taken as the maximum moment curve and the girders were proportioned accordingly.

The following stresses were used in the design for the shell of the stack:

Axial tension	10 000 lb. per sq. in.
Axial compression	9 000 " " " "
Rivet shear	10 000 " " " "
Rivet bearing	20 000 " " " "

Rivets in horizontal and vertical seams were computed at 70% efficiency. The opening in the stack for the flue has been heavily reinforced by angles and plates to compensate for the reduction in area.

To obtain the zero line (Fig. 19), Compression Areas 1 to 9 and Tension Areas 10 to 13 are laid off to a convenient scale and any pole distance, V, is taken. Draw the funicular polygons, I_c and I_t. The sides of these polygons are prolonged until they intersect the resultant, P. With these intercepts and the pole distance, V, the funicular polygons, II_c and II_t, are drawn. Their intersection point will locate the zero line.

With $f_x = b\,x$,

$$Z = \frac{\int x^2\,d\,I}{\int x\,d\,A}$$

The maximum compression in rim is,

$$f_c' = \frac{P\,.\,c}{2\,S\,V}$$

The maximum tension in anchors is,

$$f_t = f_c\,\frac{t}{c}$$

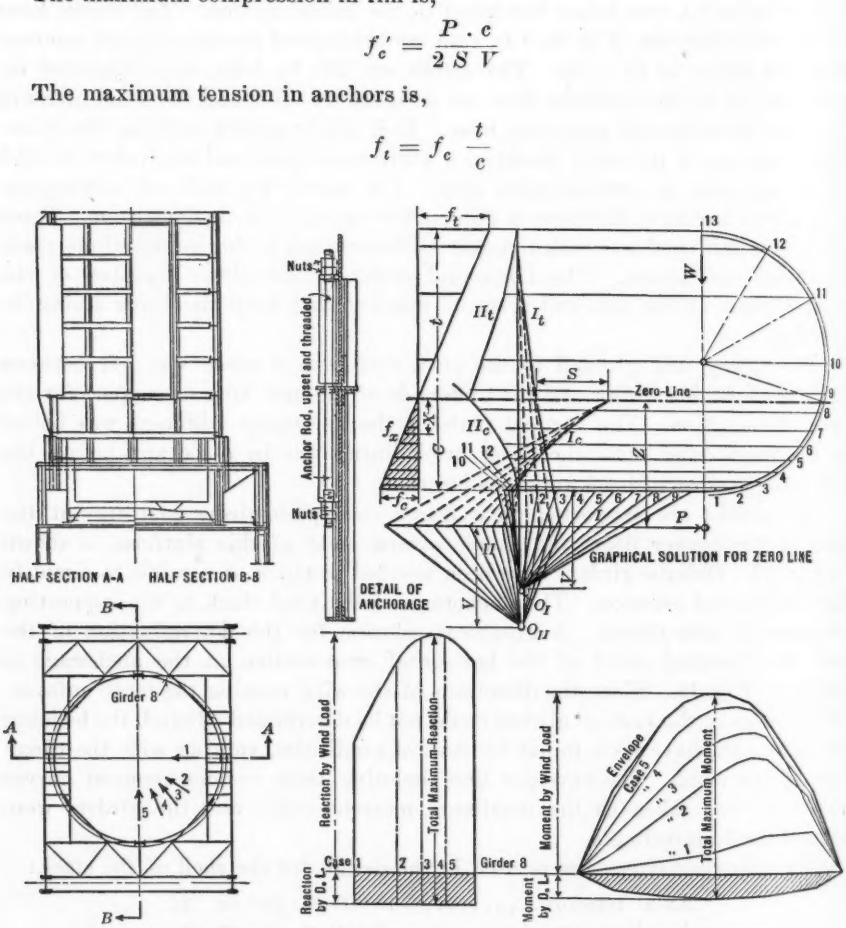


FIG. 19.—DETAILS OF STACK SUPPORT AND ANCHORAGE, KEARNY STATION.

Each group of six boilers is provided with one double-hopper, 25-ton, coal-weighing larry (Figs. 15 and 18), traveling along the firing aisle, receiving coal from the bottom of the bunkers, and delivering it to the stokers under the boilers. In addition to the weigh larry a movable platform (Fig. 15), which serves a group of six boilers, has also been provided. This platform is designed to travel the entire length of the firing aisle and is adjustable in height to suit the requirement of the work at hand on the boilers. The runways for this platform are bracketed from the overhead steel above the firing aisle. The ashes from the boilers after passing through the clinker

grinders on the stokers are dropped into a sluiceway and then delivered into a settling pit whence they are removed as will be described subsequently.

Since adequate bracing for the stacks was necessary, it was made the nucleus of the entire bracing system for the station, as may be seen from Fig. 3.

The turbine room and bunker building have no major bracing; they obtain their stability from the boiler-house. Due to boilers and stokers projecting into the firing aisle, the transverse bracing had to be brought into the vertical plane of the rear boiler columns. This necessitated the arrangement of a horizontal bracing system at the level of the stack support, in order that the wind loads acting on the stack might be transmitted into the transverse bracing system.

The boiler-house roof is carried by three-hinged trusses (Figs. 15 and 17). This three-hinged system above the induced draft blower floor, was adopted after making careful studies to determine the most economical solution of this problem, and also to simplify the installation of the flues and induced draft equipment. This design is lighter than conventional straight bottom chord roof trusses, showing a net ultimate saving of 110 800 lb. of steel for this portion of the work. Three-hinged arches are statically determined in regard to outer forces. Reactions and stresses are easily found, and the latter are not affected by changes of temperature. The arches provided an open space, free from obstructions, and resulted in obtaining a spacious room for the equipment at a reduction in the height of the building of more than 4 ft. and in the cubic contents of the buildings by 27 690 cu. ft. per section, or an ultimate total reduction of 138 450 cu. ft. of building volume.

The boiler-house building is 167 ft. 6 in. wide, 206 ft. 3 in. long for the initial installation and 495 ft. 2 in. for the ultimate station, which will consist of five units each containing six boilers, one stack, and auxiliary equipment.

Turbine Room

The turbine-room building (Figs. 15 and 20) was designed to take the following equipment:

(*a*) Three General Electric Company, 39 200-kv-a. turbo-generators, 1800 rev. per min., designed to operate at a steam pressure of 325 lb. gauge and superheat of 271° Fahr. at the throttles; and

(*b*) Two Westinghouse Electric and Manufacturing Company's 43 750-kv-a., turbo-generators operating under the same conditions.

These turbo-generators are supported on structural steel foundations. Each is equipped with one surface condenser having 50 000 sq. ft. of tube surface, the water-boxes having vertical diaphragms so that one-half of the condenser can be cleaned while the other half is in operation. The condensers weigh 700 000 lb. (full) and are supported on springs. Special rubber expansion joints are inserted in the water inlet and discharge pipes.

For each condenser two circulating pumps (Fig. 21) are installed, each having a capacity of 38 000 gal. per min., and being driven by one 500-h. p., 2 300-volt motor. In addition to these pumps, two motor-driven 1 000 gal.

per min. condensate pumps and two twin two-stage steam-jet air pumps are installed for each condenser.

Each generator is equipped with one Griscom-Russell, two pass, "U-Fin" cooler for cooling the air for the generators. The coolers for the General Electric units have a tube surface of 1 820 sq. ft. and a fin surface of 18 240 sq. ft., while the Westinghouse units are provided with air coolers having a tube surface of 2 280 sq. ft. and a fin surface of 22 800 sq. ft. each. The larger size of the latter coolers is due to the greater generator capacity of the Westinghouse machines.

To secure the greatest economy, motor-driven auxiliaries are used throughout, except the minimum spare equipment. For feed-water heating purposes a four-stage bleeding system from the main units has been installed. All possible precautions have been taken for the proper operation of the heat balance system including the bleeding of steam from the main units. To accomplish the desired results, one high-pressure, one intermediate-pressure, and one low-pressure, closed, feed-water heater complete with all necessary valves, piping, controls, etc., was installed for each unit.

For the erection and operation of the turbine room equipment two 125-ton Cleveland Crane and Engineering Company's cranes were installed (Figs. 3 and 20). The runway girders supporting the crane were designed for the two cranes in closest position with a vertical live load impact of 25% and a horizontal impact of 10% acting along the longitudinal axis of the crane. The impact in the direction of travel of the crane, due to sudden stops, was assumed at 14% of the maximum total reaction of the crane, governing the design of the longitudinal bracing in the side-walls of the turbine room. This bracing of the initial portion was placed in only three bays for structural economy and in order not to detract from the architectural appearance. Wherever possible the bracing was placed in the panel bays. Architectural demands also governed the shape of the roof trusses, which were designed with arched bottom chords (Fig. 3), thus lending to the gracefulness of the turbine-room interior.

The dimensions of the turbine room were governed by the space required for the installation and operation of the main equipment. The building is 106 ft. 8 in. wide, 283 ft. 8 in. long for the inital installation, and will be 543 ft. 0 in. long after the ultimate ten generating units are installed. The main operating floor is 35 ft. 0 in. above grade, or basement, level, while the turbine-room cranes are supported on girders 41 ft. above the main floor, or 76 ft. above the basement.

The structure of the turbine room receives its stability from the bracing system of the boiler-house. The columns of the turbine room do not line up with those of the boiler-house and the transmission of the horizontal forces was accomplished at the blower-floor elevation (or 78 ft. 0 in. above the basement) by a horizontal plate girder, riveted to the beams of the blower floor, and in part by the roof trusses (Fig. 3).

The turbo-generators are supported on individual structural steel foundations 35 ft. in height; these foundations are entirely isolated from the main building steel (Fig. 21). The design of the turbine supporting structures

FIG. 20.—TURBINE ROOM INTERIOR DURING INSTALLATION OF EQUIPMENT: (1) SURGE AND COOLING WATER TANKS; (2) SERVICE TANKS; (3) BOILER-HOUSE PIPING; AND (4) CONDUITS FOR TURBINE INSTRUMENT BOARD.

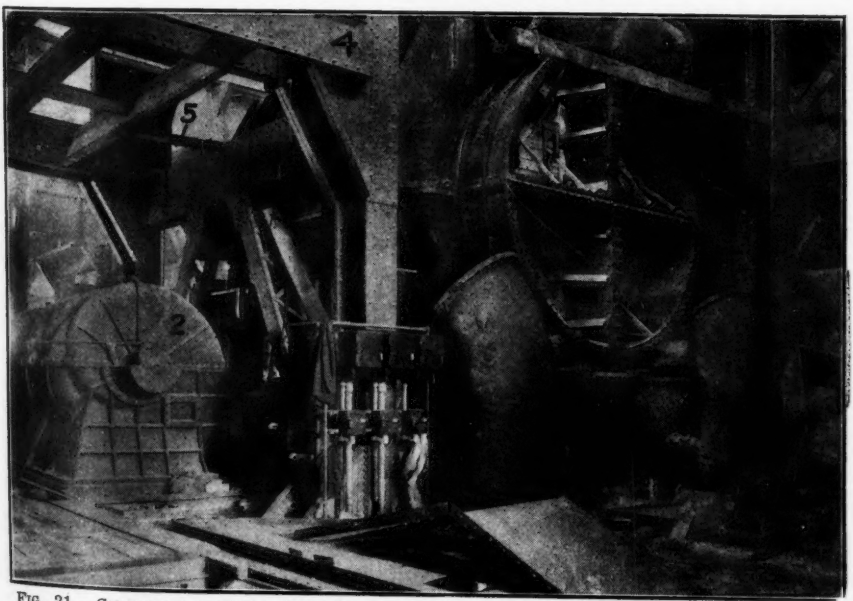


FIG. 21.—CIRCULATING PUMP CONDENSER AND PIPING: (1) CONDENSER; (2) CIRCULATING PUMP; (3) CONDENSER OVERFLOW PIPE; (4) TURBINE FOUNDATION STEEL; AND (5) SUPPORT FOR ENCLOSED GENERATOR AIR COOLERS.

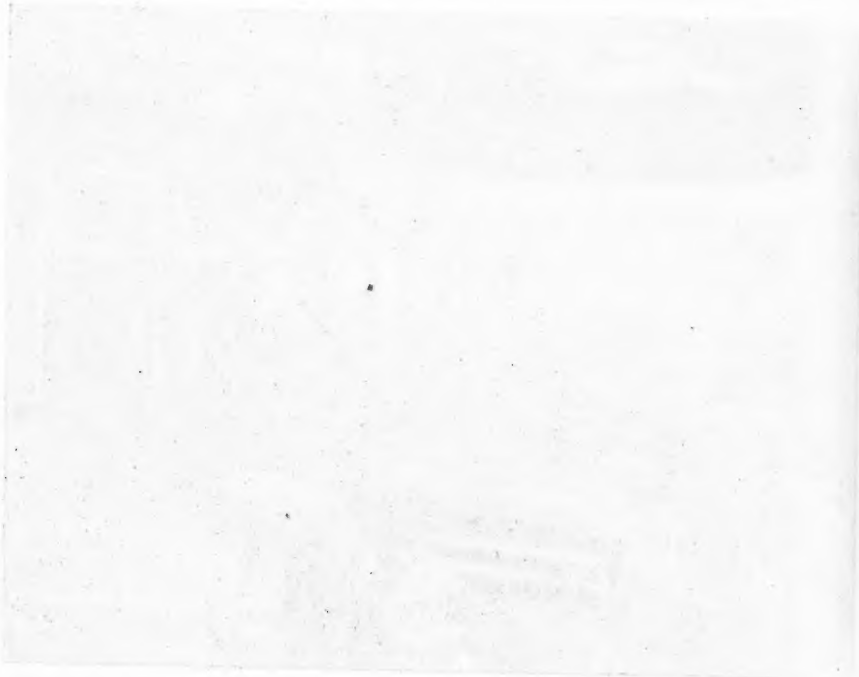


FIG. 22.—SUSPENSION OF FLOORS IN MAIN AUXILIARY SUB-STATION: (1) SUSPENSION TRUSS; (2) HANGER; AND (3) CONDUITS IN FLOOR AND WALL.

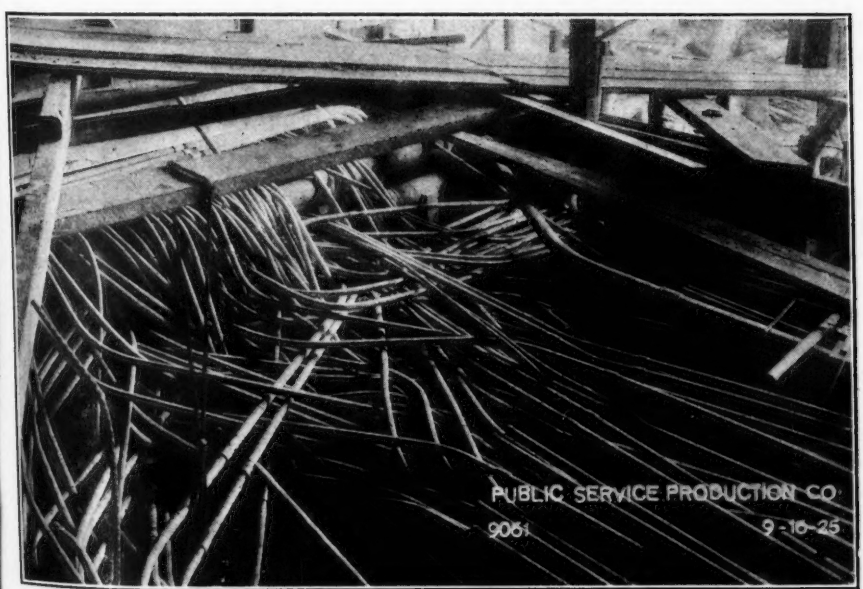


FIG. 23.—CONDUITS IN FLOOR OF MAIN AUXILIARY SUB-STATION.

was based principally on deflections, care being taken that the deflection along the longitudinal axis of the foundations was uniform under each bearing. All connections and web sections were properly proportioned for impact. The horizontal bracing was designed by assuming 30% of the vertical loads as acting horizontally.

The entire upper platform of these turbine foundations was reinforced against vibration by the addition of diaphragms and concrete fill between parallel members. The design of the foundation steel was complicated by the space requirements of the condensers, pumps, air coolers for the generator, piping, generator leads, etc., thereby limiting the space available for bracing.

In addition to the turbo-generator equipment, the turbine-room building contains the main auxiliary distribution sub-station, situated under the main floor of the turbine room and consisting of three-phase, 2 300-volt, and 440-volt busses, which are supplied from the main tie bus in the switch-house through 5% reactors, and of three 7 500-kv-a. transformer banks. Two boiler-house and three coal-handling sub-stations are at present supplied from the main auxiliary sub-station, which ultimately will supply four boiler-house and four coal-handling sub-stations. All auxiliaries have duplicate feeds and panels of the truck type. The equipment of this main auxiliary sub-station is located on three floors, the upper one carrying also the leads from the generators to the switch-house on separate supports. All sub-station floors are suspended from a triangular truss (Figs. 3 and 22) which also serves to brace the lower portion of the outside turbine-room columns. An idea of the work of conduits in the floor of this sub-station may be gained from Fig. 23.

The system of intake and discharge tunnels, suction chambers, discharge chambers, etc., complicated the foundation design both for the building and equipment. The designs, however, were developed so that the walls of the various chambers were utilized to stiffen the entire structure and consequently operated to reduce the thickness of the mat (see Fig. 12).

Intake Tunnels and Discharge Canal

The location of the station on the Hackensack River gives it an unlimited supply of condensing water. This water enters the intake tunnels through trash racks, is cleaned by revolving screens installed in the bunker-house, and then passes to the suction chambers and through circulating pumps into the condensers. From the condensers the water goes into the discharge chambers and then overflows into the common discharge canal, which delivers the warm water to the river. An individual intake tunnel, suction chamber, and discharge chamber are provided for each turbine unit (Fig. 24).

Each intake tunnel was designed to furnish 78 000 gal. of water per min. for condensing purposes and 10 000 gal. per min. for miscellaneous other station requirements, at a velocity of 3.2 ft. per sec. The discharge tunnel is designed to take care of the entire condensing water from ten units, or 780 000 gal. per min., at a velocity of 3.8 to 7.5 ft. per sec., depending on the tide.

The Hackensack River is subjected to variations of flow and of tide. To insure a continuous water supply irrespective of the tidal stage, the underside

of the roof of the intake tunnels was fixed at 2 ft. below lowest tide (Fig. 10). The ultimate arrangement is to return the water to the river at two points, one up stream and the other down stream from the intake tunnels (Figs. 2 and 24), so that the danger of re-circulation may be eliminated. Ultimately it is planned to discharge the water into the river corresponding to the direction of the tide. This is accomplished by the introduction of balanced segmental gates as shown in Fig. 24. The discharge canal terminates at the face of the dock in a forebay, which was designed to reduce the velocity of the water emerging from the tunnel, thereby facilitating the handling of barges along the dock front. The forebay was roofed over to support loadings due to 10-ton trucks over the area along the dock.

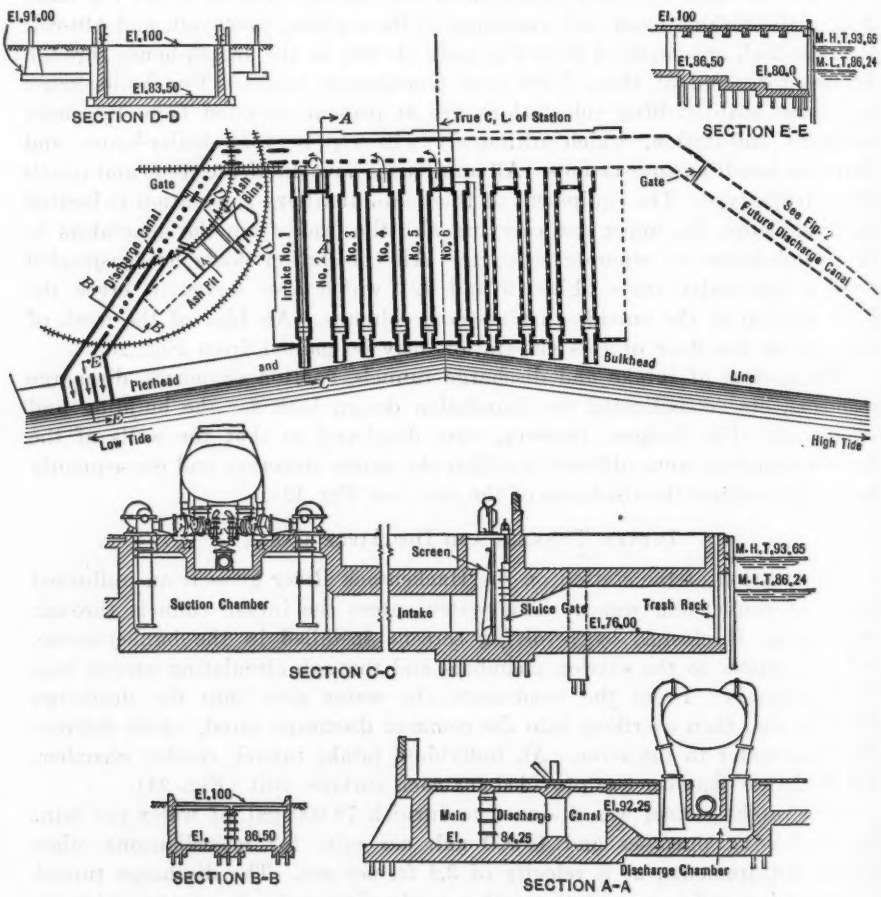


FIG. 24.—DETAILS OF CIRCULATING WATER SYSTEM, KEARNY STATION.

The intake tunnels are built of reinforced concrete and extend from the front of the dock under the bunker and boiler-house to the suction chamber under the turbine foundations. The face of the intake tunnels was brought

inward about 4 ft. from the edge of the dock to prevent obstruction to the flow by barges tied to the wharf. The rectangular cross-section of the intake tunnels is 6 ft. 6 in. wide and 8 ft. 6 in. high.

The position of the tunnels below the foundation mat was utilized to make them an integral part of the foundations of the turbine room, boiler-house, and bunkers. The vertical walls were designed to act as ribs, reinforcing the mat and allowing a safe distribution of the heavy concentrated loads over the proper number of piles. The tunnels are provided with trash racks at the entrance and revolving screens inside the bunker basement (Fig. 10). The screen wells can be shut off for cleaning and necessary repairs by a motor-operated sluice-gate.

The discharge chambers spill the water into the main discharge canal over a weir (Fig. 24, Section *A-A*). This was necessary to insure that the overflow, or tail-pipes, from the condensers should always be submerged to maintain the siphon effect, irrespective of the water level in the discharge canal, due either to the quantity of water handled or the condition of the tides. The opening above the weir was made of sufficient size to reduce the velocity of the water and to minimize the effect of erosion.

The bottoms of the discharge chambers directly below the condenser overflow as well as the top of the weir were protected against possible erosion from the condenser discharge water by heavy cast-iron plates set in place. Cast-iron plates were also inserted under the pump suction.

The discharge canal is of rectangular cross-section and extends along the entire northern side of the turbine room. The foundation walls for the main building as well as the foundations for the turbo-generators, were utilized as side-walls for the discharge canal, requiring only a slight increase in depth for obtaining the required area. To distribute the heavy turbine-room loads over the piles, the floor of the discharge canal was heavily reinforced. The roof is a concrete slab supported between heavy steel girders and carrying railroad tracks and sub-station equipment (see Fig. 12.)

Beyond the turbine room the bottom of the discharge canal was raised and the rectangular cross-section was changed to maintain the net area required. This proved more economical, since it reduced the depth of excavation and also the height of the outside walls. The construction of the discharge canal outside the turbine room is similar to that inside, including a concrete roof supported on steel beams, reinforced to take care of railroad and truck loading, whenever they occur. The roof structure, except for railroad crossings, was supported in the center of the canals by concrete piers cast inside vertical 30-in. tile-pipe spaced 15 ft. centers (Figs. 24 and 25). The pipe was used as a form for pouring the concrete and was left in place to protect the piers against erosion. At railroad crossings regular concrete piers are used (Fig. 24).

Switch-House

The switch-house structure and foundation were designed to accommodate vertically isolated phases on the three lower floors, and complete separation of oil-immersed apparatus; also manual gang-operated disconnecting switches,

and solenoid-operated oil circuit breakers with a rupturing capacity of 1 500 000 kv-a., all mechanically and electrically interlocked. The operating mechanism is on the fourth floor, and the main control room on the sixth floor, with the conduit room below.

The requirements imposed many complications in regard to obtaining a stable and suitable structure. This can readily be seen in Fig. 26. The entire central portion of the first four floors of the building is occupied by the cell structures for busses, circuit breakers, reactors, mechanism, etc. One rigid requirement was that no steel should be used which would cross through the floors from one side of the building to the other; this was to insure the greatest effect of the electrical isolation of phases.

The lowest main steel crossing the building is on the fifth floor, which carries the main storage batteries for control purposes, motor generator sets, machine and repair shops, storerooms, toilets, and conduit room. The total loading of this floor, due to these items and to the large number of conduits embedded in concrete, amounted to 375 lb. per sq. ft. This floor is suspended by hangers from the roof truss, except in those bays containing the control and conduit room. The suspended floor girders are connected by brackets to the outside columns for stiffeners (see Figs. 3 and 26).

The entire building is designed as a shell surrounding the inner cell structures for electrical equipment, and is independent of these structures for stability. For the three lower floors the stability of the building was obtained by the introduction of secondary columns opposite every second outer main column, and by the use of a horizontal bracing system at each floor level (Fig. 27). This also reduced the amount of steel required and facilitated erection.

The control of all apparatus and the operation of the generators are supervised from the main control room on the sixth floor in the center of the future complete building. The complexity of the main control system necessitated thousands of conduits, and made it essential to include provisions for terminating the conduits at or near the switchboard. Therefore, a conduit room, or vault, was built under the main switchboard for properly manipulating the conduits (Fig. 28). The control room floor is carried by the top chords of the trusses spanning across the width of the building, while the conduit floor underneath is carried on the bottom chords of the same trusses (Figs. 28 and 29).

The foundation for the building consists of a reinforced concrete mat proportioned to distribute the weight over the piling. The cell structures are supported on transverse walls, which also allowed a simple and economical installation of conduits and cables. All floors were reinforced with a minimum amount of steel; great care was exercised to isolate this reinforcement in order to prevent undesired magnetic fields. All precautions were taken to isolate the phases as perfectly as possible and to minimize all disturbances, thus assuring a safe, continuous operation of the plant.

The generator leads were brought over from the turbo-generators in concrete compartments cast integrally with the roof slabs of the bridges between

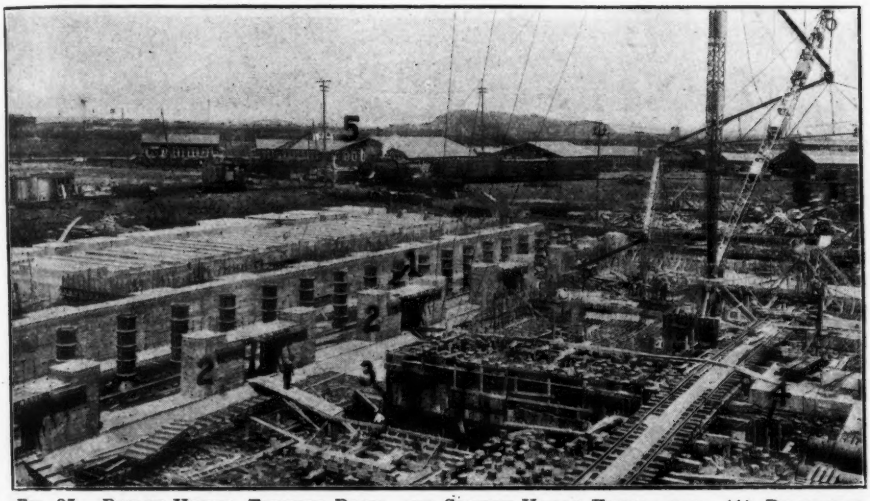


FIG. 25.—BOILER-HOUSE, TURBINE ROOM, AND SWITCH-HOUSE FOUNDATION: (1) DISCHARGE CANAL; (2) PIERS FOR BUILDING AND TURBINE FOUNDATION COLUMNS; (3) SUCTION CHAMBER; (4) INTAKE TUNNEL; AND (5) CONSTRUCTION OFFICES, ETC.

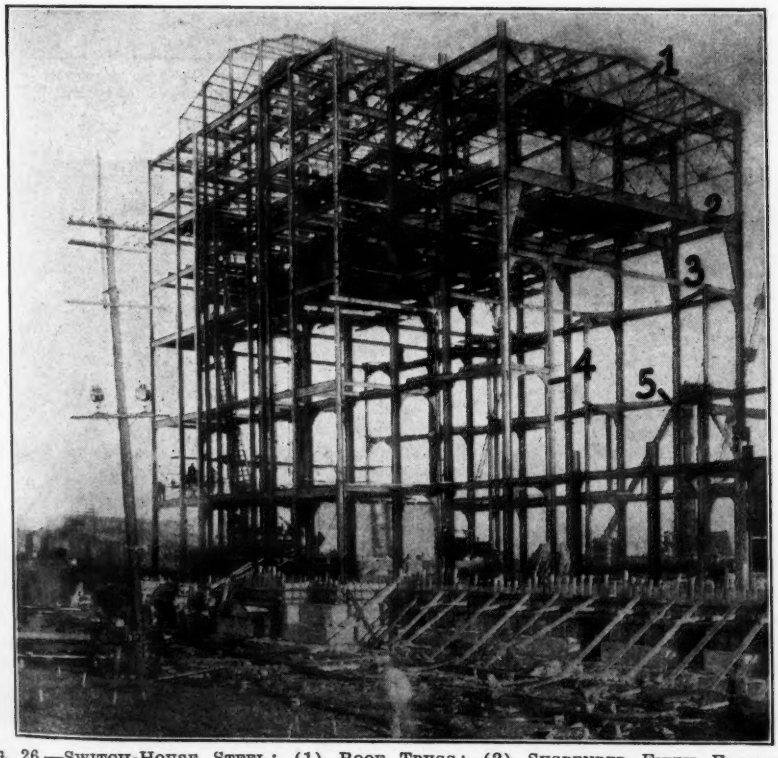


FIG. 26.—SWITCH-HOUSE STEEL: (1) ROOF TRUSS; (2) SUSPENDED FIFTH FLOOR; (3) TEMPORARY TIE; (4) VERTICAL PORTAL BRACING; AND (5) HORIZONTAL BRACE.

FIG.

FIG.

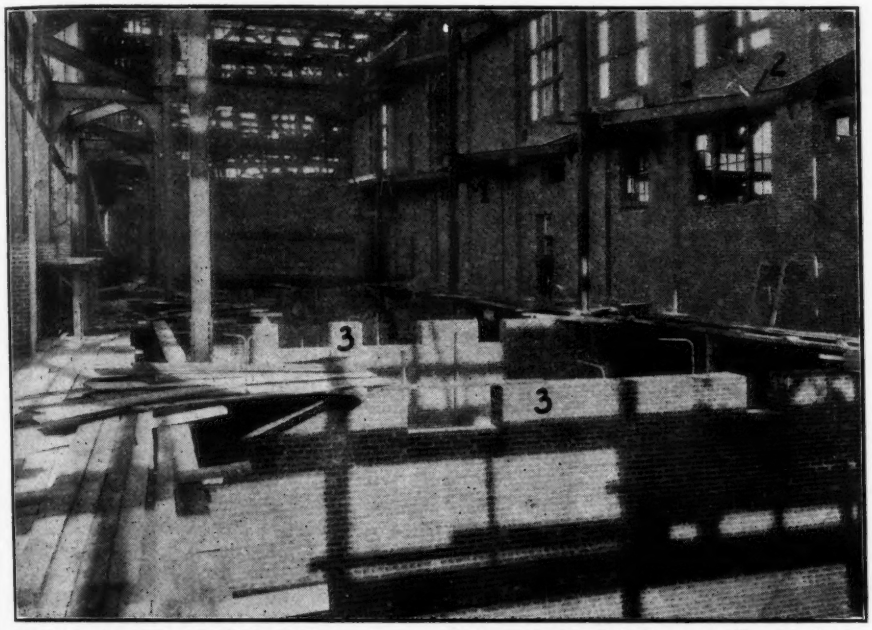


FIG. 27.—INTERIOR OF SWITCH-HOUSE DURING CONSTRUCTION: (1) VERTICAL PORTAL BRACING; (2) HORIZONTAL BRACING; (3) SECTION WALLS, STRUCTURAL. NOTE ABSENCE OF CROSS-TIES.

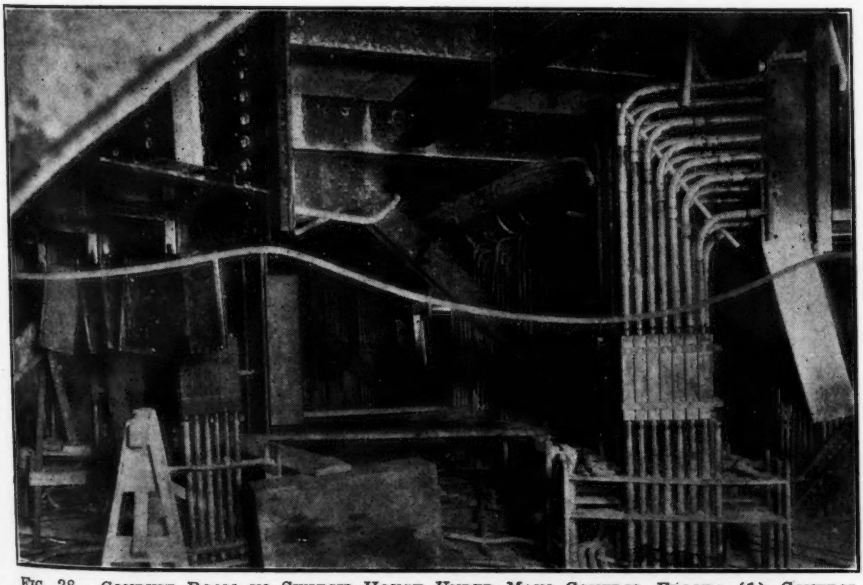


FIG. 28.—CONDUIT ROOM IN SWITCH HOUSE UNDER MAIN CONTROL FLOOR: (1) CONTROL ROOM FLOOR; AND (2) DIAGONALS OF TRUSS CARRYING BOTH FLOORS.

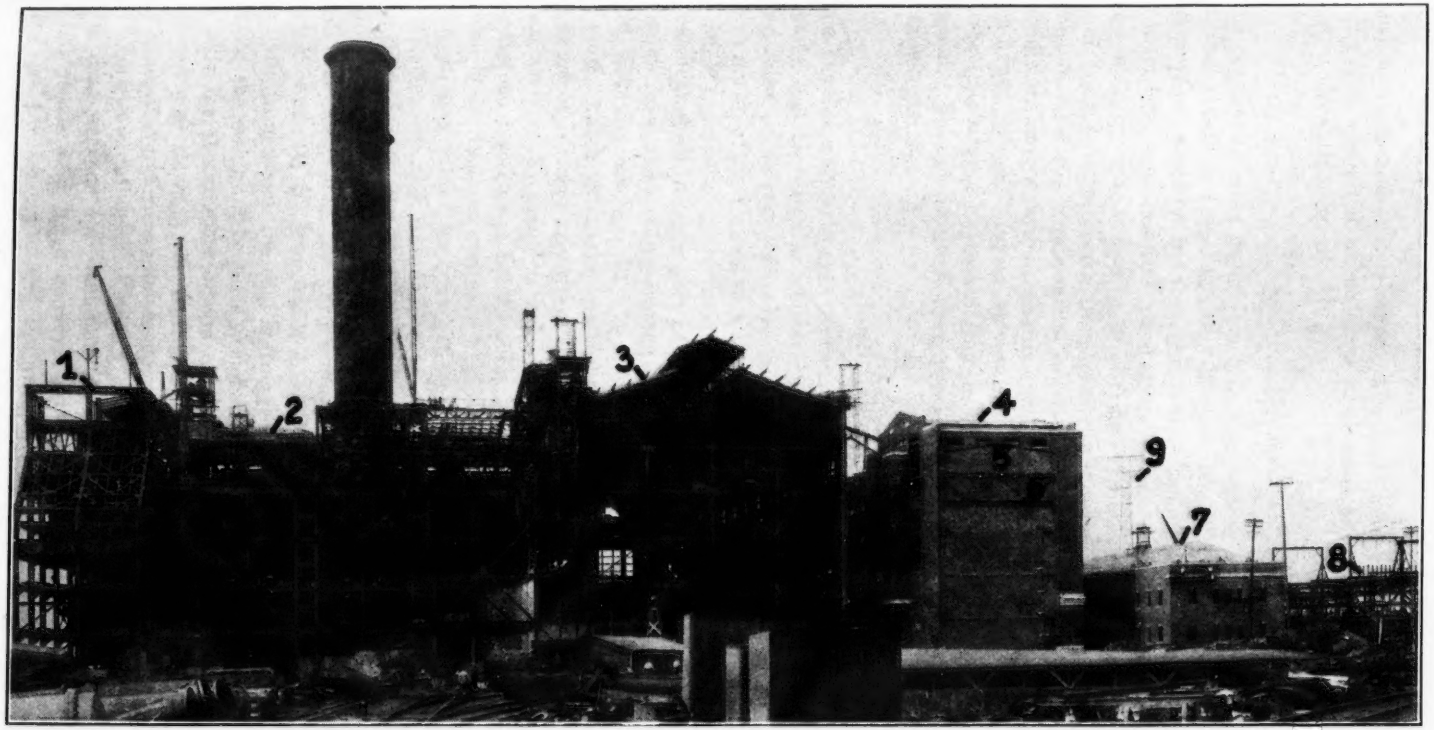


FIG. 29.—KEARNY POWER STATION, GENERAL VIEW: (1) COAL BUNKER; (2) BOILER HOUSE; (3) TURBINE ROOM; (4) SWITCH HOUSE; (5) CONTROL ROOM; (6) CONDUIT ROOM; (7) SERVICE BUILDING; (8) OUTDOOR SUB-STATION; AND (9) TRANSMISSION TOWER.

the turbine room and switch-house. The leads were carried in inverted concrete troughs supported on separate steel work through the auxiliary sub-station in the turbine room.

Service Building

The service building is designed as the central architectural feature of the ultimate station (see Fig. 4). It was planned to house all the administrative offices required for the operation of the station, as well as a restaurant, assembly room, hospital, paymasters' offices, and other minor facilities.

The building is a two-story, steel-frame structure with exterior walls of common brick, trimmed with limestone. It is carried on a pile foundation designed to meet the requirements imposed by the structure itself as well as the conditions due to various conduit and duct lines (Fig. 35) leading from the switch-house to the outdoor sub-station. These duct lines are separated from the foundations of the building, to avoid possible damage by unexpected settlement. Special features were provided to bridge the foundation walls over these conduits.

Coal Handling

The location of the Kearny Station on the Hackensack River permits the receipt of coal by water as well as by rail. A yard storage of 125 000 tons was provided. The pockets in the coal-bunker have a capacity of 3 800 tons for each group of six boilers, or a total of 19 000 tons for the ultimate station. This bunker capacity constitutes an insurance for continuity of service in case of break-down of any part of the coal-handling machinery.

The entire coal-handling system, as installed, is shown in plan on Fig. 2 and in elevation on Fig. 30. It is designed for the use of belt conveyors throughout, which gives the most satisfactory and economical service both as to construction and operation.

Coal arriving by water is hoisted from the barges by a Mead-Morrison, motor-driven, grab-bucket hoist having a capacity of 350 tons per hour; and delivered by an inclined conveyor to a Bradford breaker in the breaker-house. Thence, it may either be conveyed to the yard storage or to the bunkers. Coal arriving by rail is unloaded in the car-hopper house and dumped on the inclined conveyors leading to the breaker-house, passing through the Bradford breaker, etc., as before.

All coal designated for yard storage is carried by the conveyors to a swinging boom 90 ft. long itself having a conveyor, which builds up an initial pile from which the coal is dragged to storage by a drag scraper. The tower for the swinging boom is designed to take the loads of the boom and to provide for the support of the conveyor trestle (Fig. 30). It is tied to the drive-house through the conveyor bridge trusses, thereby stiffening the tower against overturning due to boom loads and consequently reducing the cost of both foundations and steel.

The drag scraper also reclaims the coal and delivers it to a pit, from which it is elevated by a conveyor to the horizontal reversible conveyors between the breaker-house and the yard storage (Fig. 30). These, in turn,

transfer the coal to the breaker-house and thence to the inclined conveyors leading up to the bunkers, where the coal is distributed to the various pockets as required.

The entire conveyor-storage and reclaiming system is designed to handle 400 tons of coal per hour, duplicate belts in the initial installation being provided over the bunkers and from the breaker-house to the bunkers. Provisions are made throughout for duplicate conveyors for the ultimate installation, except the conveyor from the coal tower to the breakers, the conveyor on the swinging boom, the belt feeder in the reclaiming pit, and the conveyor from the pit to the reversible conveyors.

The drag-scraper equipment, furnished by the Maine Electric Company, has a maximum capacity of 400 tons per hour, and is driven by a 200-h.p., General Electric motor, working on a maximum haul of 600 ft. and giving a yard storage of 125 000 tons for a pile 15 ft. deep. Future extensions to the coal-storage system will provide for an ultimate capacity of 275 000 tons, while an additional 90 000 tons of coal can be stored and reclaimed by locomotive cranes.

The design of the trestles, bents, and other structures was governed to a great extent by tunnels, railroad clearances, ash-handling equipment, roadways etc. These conditions made it necessary to utilize long spans for the trusses. In order to decrease the weight of steel and reduce deflection, these long spans were built with cantilever arms, 20 to 25 ft. in length, and with suspended spans 100 ft. in length (Fig. 30). The main trusses were built up of "star" web sections and channel or angle chords. The flooring consists of 3½-in. reinforced concrete, and the roof and siding of asbestos-protected metal.

The bents throughout are designed with vertical upper posts, and tower legs spreading out at the bottom (Fig. 30, Section *D-D*), thus providing for simpler shop details and presenting a more attractive structure. Due to the height of the conveyor galleries between breaker-house, coal tower, and coal-bunker the bents for these spans were built of latticed angles. The trusses were supported on both columns of the bents and made statically determined by the omission of the diagonal in the end panel (Fig. 30).

To avoid secondary stresses in these bents, the bottom diagonal was merely bolted during erection and not riveted in place until after the full load was on the bents. Wherever possible the concrete footings for the columns of these bents were tied together with concrete ribs to insure against spreading or unequaled settlement (see Section *D-D*, Fig. 30). Provisions were made in the design of the trusses and bents for expansion and contraction due to temperature changes.

The breaker-house (Figs. 30 and 32) is 54 ft. wide and 50 ft. 6 in. long by 90 ft. high. It was designed for an ultimate installation of two Bradford breakers, each 12 ft. 0 in. in diameter, 22 ft. 0 in. long, 102 000 lb. in weight, and having a capacity of 400 tons per hour. These breakers are driven by one 150-h.p. General Electric motor. In addition to spouts and transfers for coal, space for a distribution sub-station has been provided in this building.

The coal is delivered to the breaker by the conveyors from the car hoppers or coal tower, thence by a bifurcated spout to the belts conveying it either to

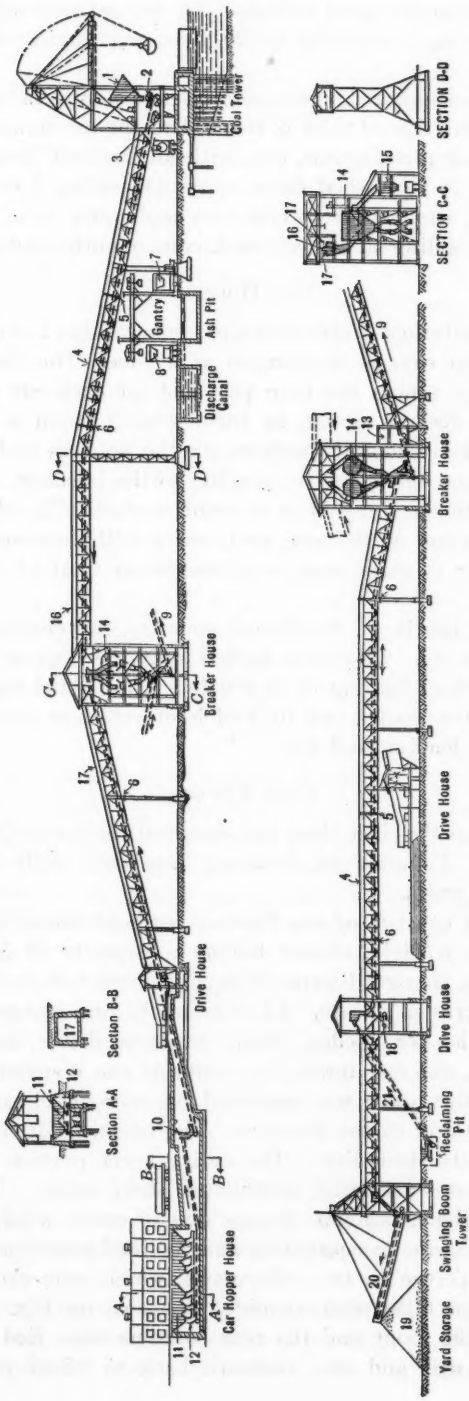


FIG. 30.—CROSS-SECTION OF COAL-HANDLING SYSTEM, KEARNY POWER STATION.

the coal-bunkers or to the yard storage. All refuse material is dumped into bins, each bin having a capacity of 20 tons, and arranged to empty into cars for disposal.

The breakers are supported on a separate set of colums in order to minimize vibrations. Accumulation of dust is reduced to the minimum by covering all horizontal projecting girts, beams, etc., with an inclined flashing. To permit the concrete floors to be washed down, curbs projecting 4 in. above the floor level are furnished, which prevent coal dust and water from getting into the joints between the siding and steel, and, consequently, reduce corrosion.

Car Hopper

The initial installation of the car-hopper house (Figs. 2 and 30) and unloading pit permits four cars to be dumped at a time. One large steel hopper with three discharge spouts has been provided for each car and delivers the coal to horizontal feeders which, in turn, deposit it on a 36-in. conveyor. This conveyor runs horizontally underneath the hoppers and then rises to a spout delivering it on the conveyors leading to the breakers, from which it is transferred either to the bunkers or to yard storage. The design allows for a duplicate installation at the west end, which will increase the capacity of this unit from four to eight cars, or a maximum total of 800 tons of coal per hour.

The unloading pit is of reinforced concrete, water-proofed throughout (Fig. 30, Section *A-A*). The track girders over the hoppers are designed to take a maximum wheel loading of 25 000 lb. The tunnel beyond the hopper building is also water-proofed and its roof is of sufficient strength to take the maximum railroad loading and fill.

Coal Tower

The coal tower on the river-front was designed for the initial installation of one hoisting unit. Foundations, however, have been built for the ultimate installation of two units.

Each tower unit consists of one 350-ton per hour Mead-Morrison hoisting machine, with one 5 500-lb. bucket having a capacity of 5 000 lb. of coal, driven by a 300-h.p., General Electric Company, 2 300-volt motor, and designed to make a round trip in 26 sec. In addition to one motor-driven air compressor, receiving hopper, feeder, chute, conveyor drives, and other miscellaneous equipment, the coal tower also contains one distribution sub-station.

The height of the tower was governed by railroad clearances under the conveyor trestle leading to the breakers. The boom, 60 ft. long, is arranged to swing clear of the dock line. The entire lower portion of the tower is enclosed with asbestos-protected, corrugated sheet metal. The foundations were designed for the maximum stresses due to snow, wind, bucket-loading, dead loads, and loads due to equipment and inclined conveyor trestle.

The piling was driven in two coffer-dams, which were excavated to 30 ft. below mean tide and filled with concrete, as shown on Fig. 30. The coffer-dams under both the front and the rear columns were tied together at the top by a concrete mat and also anchored back to "dead men". This was

deemed advisable owing to the soil conditions at this point. The encasement of the piles by concrete also protects them against an eventual attack by marine borers, which are known to be in the vicinity of New York Harbor.

Ash Handling

The ashes from the boilers are washed through sluice-ways to a settling pit at the west of the building (Figs. 2 and 31).

The sluice-ways are arranged so that each group of three boilers deposits into one short continuous trough, or sluice, which leads into the main sluiceway carrying the ashes to the settling pit. An electrically operated Cleveland gantry crane, equipped with a grab-bucket, travels along the entire length of the settling pit and hoists the ashes into overhead hoppers from which they can be dumped either into trucks or railroad cars (Fig. 31). A large overhead hopper capacity was provided to make the operation of the gantry crane independent of cars or trucks for ash disposal. The pits have a storage capacity of 600 cu. yd. of ashes, which will take care of somewhat more than a maximum day's use of the ultimate station. Extensions to the pit can be made without impairing the operation, if later demands should require.

The two interior partition walls in the settling pit are made of timbers for easy removal if required for pit extensions. Overflow openings in the partitions and end walls are installed, with skimmers to prevent floating cinders from being carried into the discharge canal. The settling pit was designed to withstand hydrostatic head on the bottom, and car and truck loading on the sides. The hopper foundations were combined with the pit structure (Fig. 31).

Docks

In order to receive coal and materials shipped by water, it was necessary to build a dock along a part of the river-front of the property. With the exception of those sections in front of the bunker-house and the coal tower, the dock has been constructed of wooden piles and timber deck supporting a 9-ft. cinder fill, with a reinforced concrete retaining wall 9 ft. high. along the face. The concrete face is protected against injuries from boats, etc., by timbers and fender piles (Fig. 3).

The design of the dock was based on a dead load of 500 lb. per sq. ft., a uniformly distributed, superimposed live load of 500 lb. per sq. ft., for storage purposes, equivalent to a Cooper E-50 railroad load.

At the rear of the dock, sheet-piling was driven to prevent the earth from sliding and being washed out. To resist the resulting horizontal earth pressure a number of batter piles were provided (Fig. 3).

The Outdoor Sub-Station

The outdoor sub-station (Figs. 2 and 33) was designed to step-up the entire electrical output of the generators from 13 200 to 132 000 volts for transmission purposes.

The initial installation is: Fifteen 15 000-kv-a., Allis-Chalmers, single-phase, oil-insulated, self-cooled transformers; four 90 000-kv-a., 132 000-volt, three-phase, sixty-cycle, outgoing feeders; thirteen General Electric (type

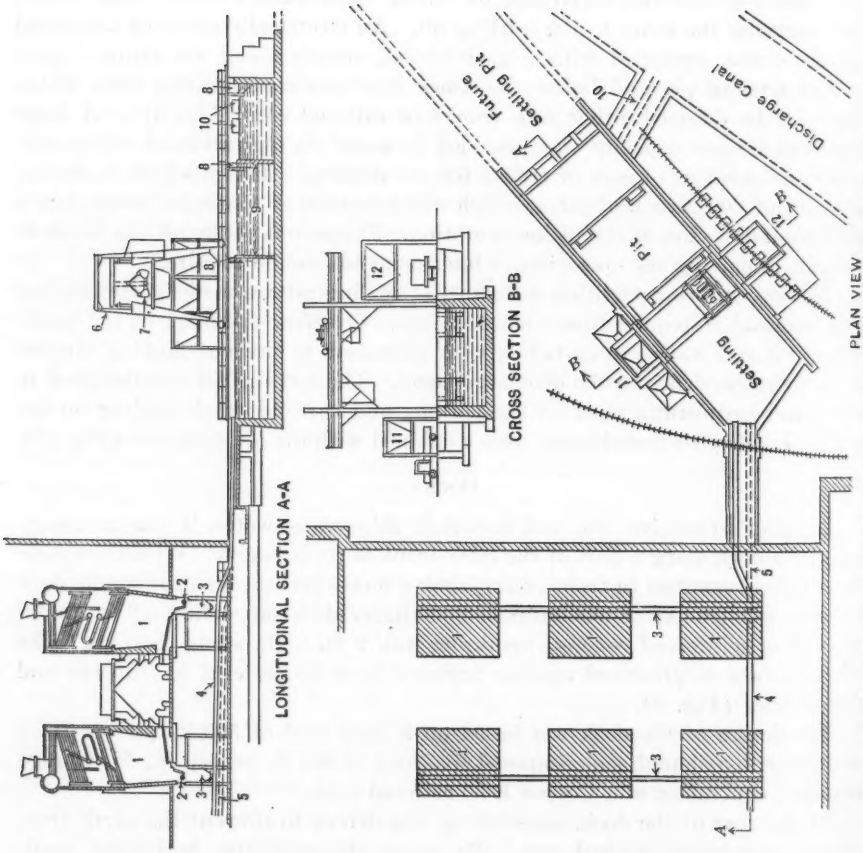


FIG. 31.—PLAN OF ASH-HANDLING SYSTEM, KEARNY POWER STATION.

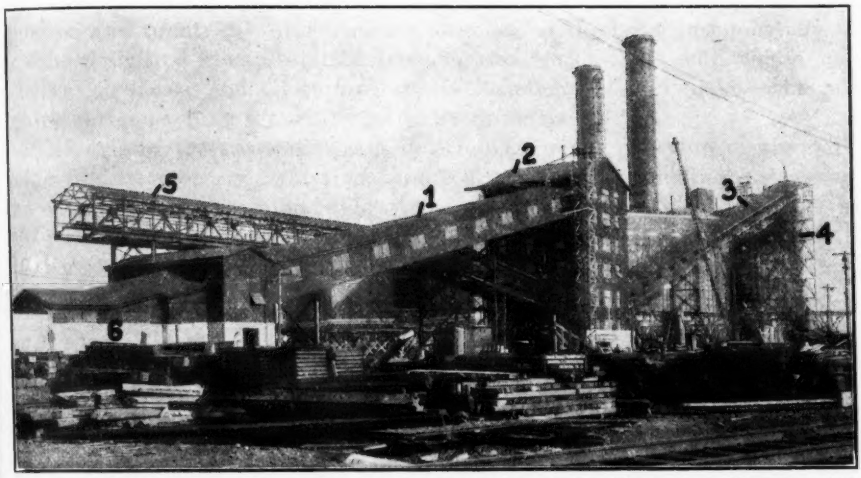


FIG. 32.—COAL-HANDLING SYSTEM: (1) CONVEYOR FROM CAR-UNLOADING HOPPER; (2) BREAKER HOUSE; (3) CONVEYOR TO BUNKER; (4) BUNKER; (5) CONVEYOR TO AND FROM YARD STORAGE; AND (6) DRIVE HOUSE.

PLAN VIEW

FIG. 31.—PLAN OF ASH-HANDLING SYSTEM, KEARNY POWER STATION.

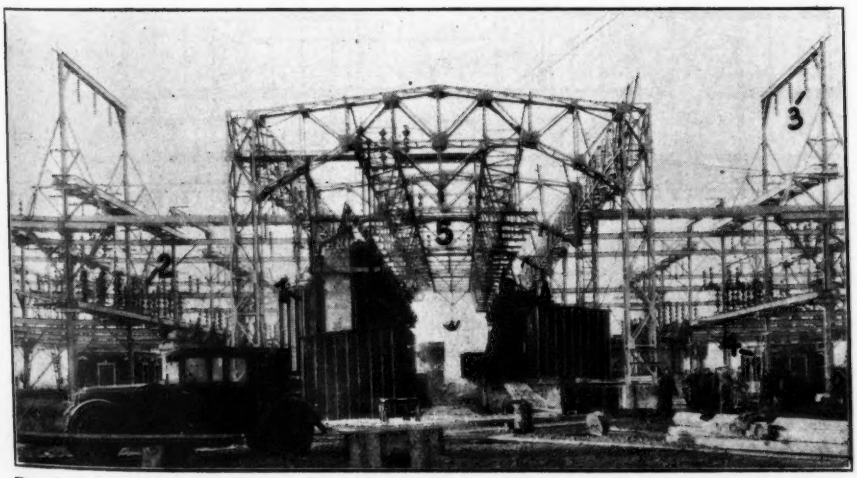


FIG. 33.—END VIEW SHOWING TRANSFORMER BAY OF SUB-STATION: (1) TRANSFORMERS; (2) DISCONNECTING SWITCHES; (3) SUSPENSION INSULATORS; (4) RELAY HOUSE; AND (5) PORTAL TRUSS OVER TRANSFORMER BAY.

"FHKO-39"), 132 000-volt, oil circuit breakers, with remote control, having a rupturing capacity of 1 500 000 kv-a.; fifty-five Westinghouse (Type 800 A), 154 000-volt, motor-operated, remote-control, disconnecting switches; thirty-three transformers; ten feeders; thirty-three oil-circuit breakers; and one hundred and twenty-five disconnecting switches, of the types just mentioned.

The design of the sub-station includes two relay houses with vaults, oil purification house, and oil storage system, together with all the mechanical equipment and piping for treating and handling the oil.

The axis of the sub-station was determined by the direction of the outgoing feeders, property limitations, and coal storage and handling system for the operation of the station. The transformers, therefore, were arranged in two parallel rows along the center line of the sub-station and the oil circuit breakers in the adjoining bays as shown in Figs. 2, 34, and 35.

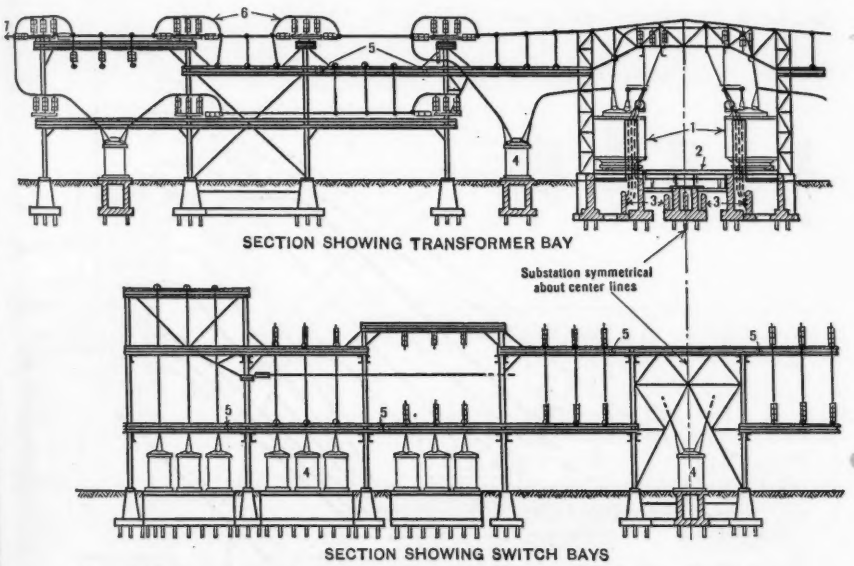


FIG. 34.—SECTIONS OF OUTDOOR SUB-STATION, KEARNY POWER STATION.

The electrical requirements as to clearances were determined at 8 ft. 0 in. between phases and 6 ft. 0 in. to ground, thereby establishing the general dimensions of the sub-station. The steelwork involved a departure from general practice by replacing latticed sections with solid shapes. H-sections were used for columns and channels for overhead supporting steel, except in the transformer aisle (Fig. 34), where the columns consist of four angles latticed, supporting an arched truss which carries the busses and the necessary apparatus for the overhead transformer connections.

The stability of the entire structure is obtained by diagonal bracing in the center or end bays (Fig. 34). The horizontal members supporting the busses and disconnecting switches consist of two 15-in. channels, braced laterally and designed as continuous beams over two or more supports with

splices at the points of contraflexure. This arrangement was found to be considerably more economical than the usual lattice sections and simplified the shop details and erection, as well as the maintenance of the structure.

The foundations for the steel and equipment are carried on piling throughout. The transformers weigh 170 000 lb. each, and their transfer for repair makes it necessary that the tracks supporting the transfer car be on a solid footing so that they can be handled (Fig. 33).

The duct runs, feeding each transformer, consist of 5-in. fiber ducts encased in concrete; one 5-in. fiber duct was provided as a spare for emergency use. These ducts are carried to the top of the transformers in concrete piers (Figs. 34 and 35). Between the sub-station and the switch-house the duct runs are supported on piles, care being taken to leave sufficient space between adjacent lines for heat radiation, and also to pitch the ducts toward the manholes for drainage. Several specially supported duct groups or conduit lines for control cable were installed.

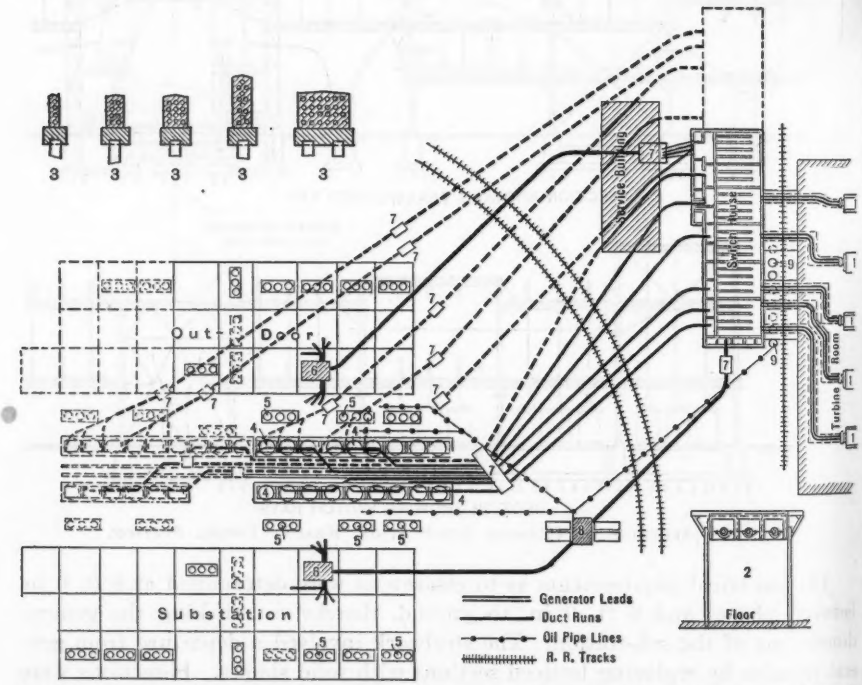


FIG. 35.—PLAN SHOWING GENERATOR LEADS AND DUCT RUNS FROM SWITCH-HOUSE TO OUTDOOR SUB-STATION, KEARNY STATION.

The transmission lines from Kearny to Marion and Essex Stations consist of a high-tension bus suspended from steel towers connecting the three stations. The tower line uses a private right of way over the "Meadows" and is designed to meet railroad and harbor regulations at railroad and river crossings. The towers are designed to carry two lines and a ground. The transmission wires are 500 000 cir. mils copper strand supported from the cross-

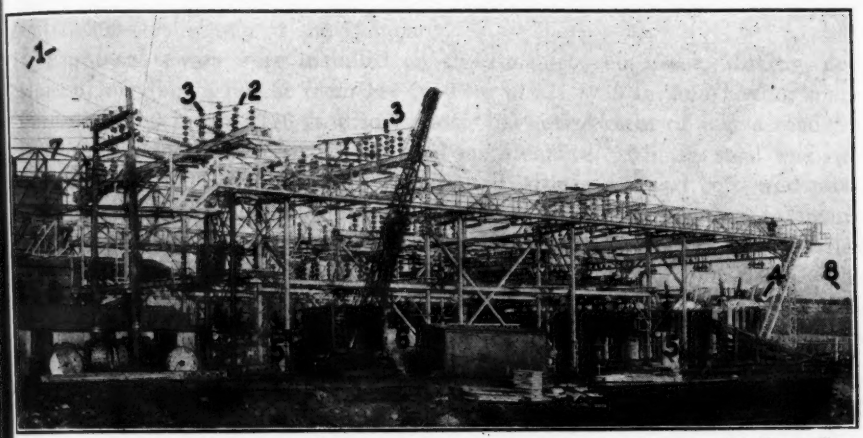


FIG. 36.—OUTDOOR SUB-STATION, MARION: (1) OUTGOING LINES TO TOWER; (2) DEAD END FOR OUTGOING LINES; (3) DISCONNECTING SWITCHES; (4) TRANSFORMERS; (5) OIL CIRCUIT BREAKERS; (6) OIL STORAGE TANKS; (7) BRIDGE FOR LEADS TO SWITCH HOUSE; AND (8) CENTER OF ULTIMATE STATION.

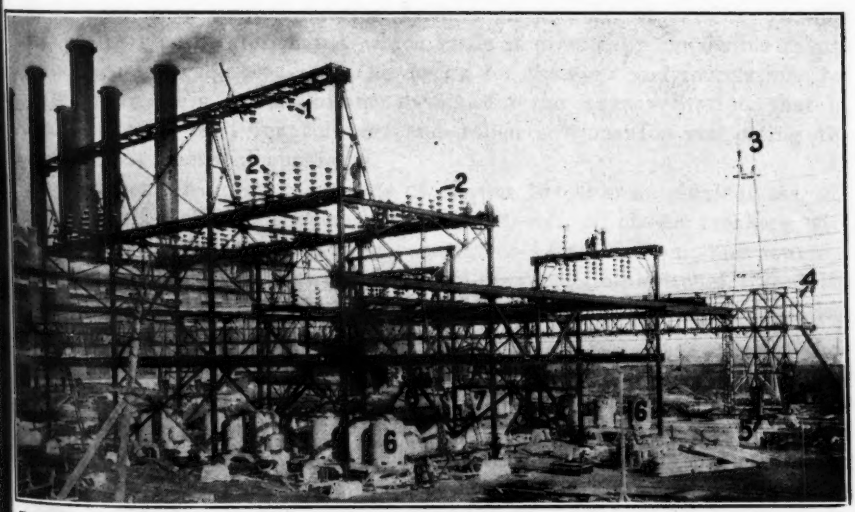


FIG. 37.—OUTDOOR SUB-STATION, ESSEX: (1) DEAD ENDS FOR LINES FROM TOWER; (2) DISCONNECTING SWITCHES; (3) TRANSMISSION TOWER; (4) BRIDGE FOR FEEDS TO SWITCH HOUSE; (5) BENT FOR BRIDGE; (6) OIL-CIRCUIT BREAKERS; (7) RELAY HOUSE; AND (8) OIL STORAGE TANKS.

arms by suspension insulators. The energy is transformed at the Marion and Essex Stations to lower voltages as desired and distributed over the 26 400 and 13 200-volt system of the Company.

Eighteen towers were installed on the line between Essex, Kearny, and Marion, varying in height from 116 ft. 0 in. to 231 ft. 6 in. and having spans from 600 ft. 0 in. to 1 136 ft. 0 in. Along the entire right of way a roadway was built to facilitate construction and maintenance. All the steel was galvanized and all field connections were bolted with galvanized bolts and nuts.

The corner piers of all tower foundations were tied together to insure stability. The foundations on the river banks where no docks are installed are protected against damage from ice by wooden sheet-pile coffer-dams. The entire space inside these coffer-dams was filled with cinders. The foundations for all towers are supported on piles, designed to sustain the loading from wind, snow, ice, wires, and pull due to angularity in the transmission line.

The tower footings for the terminal tower at Marion Station were complicated by space limitations, an angle in the line, dock construction, and the great depth to which piles had to be driven to get proper bearing. Due to these conditions it was found necessary to anchor the foundation against overturning; this was accomplished by the introduction of ties, utilizing the foundations of the building for anchorages. The gas and oil lines governed the location of these ties.

The 13 200-volt feeders from the outdoor sub-station at Marion to the switch-house were carried on steel bridges on account of existing conduit lines, and gas and water mains, which made it practically impossible to use an underground cable system. The design for the steel and foundations for this structure (Fig. 36) is similar to that for the Kearny Station, that is, H-sections were used for columns and rolled sections for supporting the miscellaneous electrical apparatus.

The present installation consists of: Seven 15 000-kv-a., single-phase, oil-insulated, self-cooled transformers; five 132 000-volt, oil circuit breakers, with remote control, having a rupturing capacity of 1 500 000 kv-a.; and fourteen 154 000-volt, motor-operated, remote-control disconnecting switches. The ultimate sub-station will have thirteen transformers, ten oil circuit breakers, and twenty-eight disconnecting switches.

The general design of the outdoor sub-station at Essex (Fig. 37) follows that of Marion and Kearny. At Essex, as at Marion, the 13 200-volt feeds for the outdoor sub-stations to the switch-house were carried on a light overhead trestle. Difficulty in spacing of bents, due to railroad tracks and other local obstructions, made it necessary to select long spans. In order to reduce the weight of the steel to the minimum, cantilevers and suspended spans were used. The length of this bridge is 508 ft. Expansion joints are provided as well as pivot bearings at the base of the bents to allow for movement due to temperature changes.

The present installation at the outdoor sub-station at Essex consists of: Seven 15 000-kv-a., single-phase, oil-insulated, self-cooled transformers; five 132 000-volt, oil circuit breakers with remote control, having a rupturing

capacity of 1 500 000 kv-a.; and fourteen 154 000-volt, motor-operated, remote-control disconnecting switches.

Future Developments

The completion of the present portion of the new Kearny Power Station, together with the outdoor sub-stations at the Kearny, Marion, and Essex Stations and the 132 000-volt interconnecting transmission lines, will form the first link of the ultimate State-wide high-tension transmission system as planned by the Public Service Electric and Gas Company. This ultimate high-tension transmission system will consist of more than 100 miles of 132 000-volt transmission lines, together with not less than nine 132 000-volt distribution sub-stations.

This high-tension transmission project will eventually spread across the entire State of New Jersey and will take the form of an immense letter P. It will provide carrying capacity for practically the entire load of the system not radially distributed from the main generating stations. The upper portion of the loop of this transmission system will include about 50% of the system's load and the tail will extend at least as far as Trenton and possibly will include Camden. This transmission system when completed will provide facilities for the location of new super-power stations at any point of the loop or system that may be determined by geographical considerations, and will make it possible to feed the entire output of these new super-power stations directly into this high-tension distribution system.

The design and the construction of the Kearny Power Station, of the Marion, Essex, and Kearny Outdoor Sub-Stations, as well as of the high-tension transmission line connecting these sub-stations, was done by the Public Service Production Company.

Acknowledgment is due to E. Rothenburg, Structural Designer with the Public Service Production Company and associated with the writer on the design of the steel and foundations of the Kearny Power Station, for valuable suggestions and assistance in the preparation of this paper.

AEROPLANE TOPOGRAPHIC SURVEYS

Discussion*

By George T. Bergen, M. Am. Soc. C. E.†

George T. Bergen,‡ M. Am. Soc. C. E. (by letter).§—Probably only limited comment is to be expected on a paper dealing with a subject somewhat new. The writer is highly gratified by the extent of the discussions in this instance and the notable sources from which they come.

Few engineers have had such extensive experience with aerial photographs as Major Fiske recites,‖ involving 13 000 film exposures and the actual use of several thousand photographs in mapping, even if this does not include their correction for tilt and their use for contour location by means of the properties of the photographs themselves. To such a method of use the paper was expressly directed.

It seems that the paper did not prevent some misconceptions regarding the method discussed. An illustration is Major Fiske's statement that the stereometer, or measuring stereoscope, gives points of equal elevation above a ground plane parallel to the negative at exposure, which would cause an error of 92 ft. in elevation 1 mile from the optical center if the plane were tilted 1 degree. In elevation determinations by the stereometer two overlapping plates are always involved, with relations as suggested in Fig. 5.¶ The operation of this instrument cannot be expressed in terms of a single photograph, but, as nearly as the writer can show, the relations are as in Fig. 14, in which the parallax would be a measure of distance normal to the tilted plane. Thus, the error would be,

$$e = A\,N - h = M \tan\theta \cos\theta + h \cos\theta - h$$

For a point in the true datum plane, $h = 0$, and if θ is small, $\cos\theta = 1$ (virtually), and $e = M \tan\theta$, which, if M is 1 mile and θ is 1°, would approximate 92 ft., as stated.

The assumptions involved, however, are faulty. They presuppose that the aeroplane and focal plane in effect glided down a 1° slope between exposures, so that both plates would be in one plane tilted 1° from the horizontal, a combination of circumstances not met in practice. They assume that the pilot lost 92 ft. in elevation from one exposure to the next, which is excessive

* Discussion on the paper by George T. Bergen, M. Am. Soc. C. E., continued from December, 1926, *Proceedings*.

† Author's closure.

‡ Pres., Bergen Eng. Service Corp., Lakeland, Fla.

§ Received by the Secretary, January 28, 1927.

‖ *Proceedings*, Am. Soc. C. E., September, 1926, Papers and Discussions, p. 1491

¶ *Loc. cit.*, March, 1926, Papers and Discussions, p. 374.

for pilots trained in connection with the method discussed, with a modern altimeter in the cockpit and conditions suitable for photographic flight. However, in correcting the pictures for tilt and conversion of the photographic record to map form, allowance is made for a variation in the height of lens as indicated by the stereometer and associated instruments from the photographs.

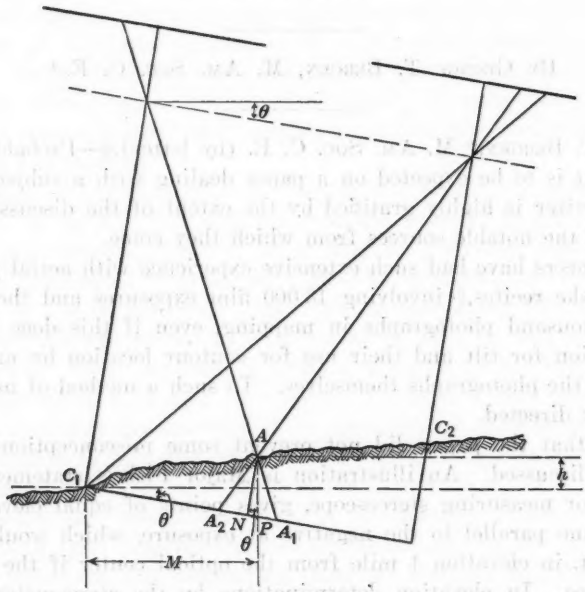


FIG. 14.—INCORRECT PARALLAX.

The most faulty assumption in Fig. 14, however, is that "the machine which indicates the contours gives points of equal elevation above a reference plane on the ground parallel to the negative at the moment of exposure". Only plates free of tilt or reprojected to the horizontal are used on this instrument for contour location so that the determinations, normal to such plane, are true elevations.

Major Fiske states that there are other mapping methods in which numerous agencies, governmental and private, use aerial photographs with results from an engineering standpoint quite as satisfactory as the one under discussion. In amplification he describes two ways of actually handling the photographs in such a process of mapping in this country. Both these methods are attributed to Government agencies, both involve all the work of contouring in the field, both compromise with the nature of photographs as perspective projections on tilted planes, and both are limited accordingly in their application.*

One of these methods is that used in recent years in some of the work on the series of topographic sheets that Major Fiske refers to as "standard

* *The Military Engineer*, September-October, 1924, p. 399; *Proceedings*, Am. Soc. C. E., March, 1926, Papers and Discussions, p. 368.

maps". He finds such maps often radically wrong in contour location and of an inaccuracy unbelievable. The implication is that this criticism does not apply to the maps of this series made with the aid of aerial photographs. Yet, in making these maps, with or without the use of photographs, the contour location is accomplished on the ground, with essentially the same instruments and methods in either case. The fact is that this great series of standard maps has been in the making over a long period of years and that during that time there has been a notable improvement in their general accuracy due to the adoption of higher standards, better control and improved organization, instruments, and practice, in the main quite independent of the use of photographs. This improvement with time is known to many engineers who, in judging the probable reliability of any particular sheet, will look at its date, rather than question whether photographs were used in its making.

Mr. Davis,* however, points out the most important factor limiting the comparative utility of the standard maps for engineering purposes, namely, their small scales, usually approximating ½ in., 1 in., or 2 in. to the mile. The aerial method discussed is of general application in topographic mapping for engineering purposes, while the small scales of the standard maps limit them to reconnaissance surveys, as Mr. Davis believes. His conclusions are reinforced by unusual experience, including prominent connection with the making of the standard maps, some contact with the new instruments for the aero-topographic process, and a distinguished career of engineering service, public and private, involving topographic mapping and its practical uses. The clear distinction as regards scale is illustrated by the practice of engineers in ordering mapping by the new process. Such orders for work in the United States are accompanied by the standard maps whenever available, which are used for the purpose of outlining the areas to be surveyed.

Mr. Davis clearly points out the great value of obtaining accurate topographic maps of large area in a shorter time than was formerly required. Such advantages of the aerial method have been well illustrated in surveys made for power projects.† This is a field of great opportunity for use of the new methods and instruments. In the successive steps of utilizing the power possibilities of a drainage basin, from the preliminary reconnaissance of a virgin field to the progressive determination of major links in the ultimate net of transmission lines, the aerial survey and its products offers advantages. The field engineer seeking sites for storage and development, the hydro-electric engineer weighing alternative plans, the transmission line engineer selecting routes, the public utility executive, the right-of-way department, and counsel may all gain in time, cost, or wealth of information.

The field engineer commencing investigation of a large drainage basin can save a tremendous amount of time by direct use of the aeroplane. Flying as an observer, he can get a broad view of the area with which he is to deal, such as is possible in no other way. He can traverse the area or follow a long watercourse in a small fraction of the time required on the ground.

* *Proceedings*, Am. Soc. C. E., September, 1926, Papers and Discussions, p. 1500.
† *Electrical World*, Vol. 86, No. 7, August 15, 1925.

Flying relatively low, he can get a good idea of the power reaches of the stream and the sections where favorable dam sites and storage are to be sought. In short, he may get quickly that grasp of the whole problem that is needed to guide intelligently more particular examination and study, and to save the time and expense required in detailed investigations. It is impossible to fix in the mind all the detail seen from the air, but by photographing from considerable altitude with a short focus lens, a valuable reconnaissance record soon may be had in the form of a photographic mosaic. This may be made as small as 1 500 or 2 000 ft. to the inch, showing 5 to 10 sq. miles in a single photograph. A mosaic of these pictures will show the water-shed and drainage lines; the timbered, cleared, and cultivated sections; the towns, roads, and other features. On it can be marked the sections needing more detailed examination on the ground and from it can be determined the best routes and program to follow in visiting these localities. On the mosaic, rough measurements can be made of all these things. In the absence of an accurate map it is invaluable.

By proper choice of altitude and lens, or by enlargement from good negatives, the photographs can be made of almost any desired scale, subject always to the deviations due to camera height, tilt, and perspective, as already set forth. These deviations can be determined and a topographic map produced by the instruments of precision and the processes described. Such a map, coupled with a mosaic of the larger scale photographs, offers the hydro-electric planning engineer a record much more complete than he has usually had. If he is dealing with a large area the mosaic is useful in defining the parts to be mapped and the order of mapping. If the whole is suitably photographed while the aeroplane is on location and a moderate amount of ground control is obtained, the files of photographs establish a resource from which additional topography may be developed, if and when later needed, without the necessity of re-organizing, transporting, and maintaining survey and mapping parties.

Mr. Faison* is one of those engineers who can speak of maps made with such means and for such purpose from detailed experience with them and with location surveys by the older methods. There are numerous factors involved in what he aptly summarizes as the "precision ratio" of the new mapping instruments. In actual practice the "height of lens" has varied from 2 400 to 17 000 ft., without fixing absolute limits and without losing photographic sharpness. The "length of lens" is equally a factor in fixing photographic scales and widens the choice. Of two sets of photographs taken for large and small scale mapping, one ran less than 100 acres per negative, while the other had more than 9.5 sq. miles on a plate of the same size. Low flying for large scale photographs tends toward increased tilt because the air is less "smooth" and the camera has less time to return when displaced from the vertical, but there are compensations in the increased sensitiveness of parallax readings and, because of the larger images, in the ease of identifying the objects. There are, of course, both high and low limits for the elevation

* *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1694.

of flight for this purpose. Probably neither absolute limit will be a matter of accuracy, but the high limit will be physical and the low limit economic, with the prescribed map scale and attendant accuracy fixing the intermediate choice for any particular case.

Mr. Faison is doubtless correct in stating that through large areas of timber it is impossible to see the ground surface continuously for considerable distances; nor is this necessary. The slopes and contours can be fixed if only frequent glimpses of the ground can be had and clearly recognized. It is this recognition of the drop from treetop to base that is aided by the vertical magnification, and the effect of distortion is avoided by using the center of the optical field. There are doubtless areas where vegetation completely hides the ground from above, but the fact is that none such has been encountered in the surveys made by the new method, which include parts of the Eastern United States that are mountainous and heavily timbered. It would not be possible to "run out" the contour lines in the equatorial jungles, nor is the writer acquainted with such work on the ground; but without penetrating the jungle the special aerial method could produce a contour map of such an area with at least barometric accuracy. Any control available at the margin of the jungle could be connected and extended by the photographs and correcting projectors. There are various interesting possibilities of applying the new methods to such country, which it is hoped may soon be initiated.

Meanwhile, Mr. Faison's statements regarding improvements of the process as already used in accurate work are very true. A number of marked improvements have been accomplished since the Green River survey that he describes. The special cameras necessary for accurate work have become available in larger number and with wider choice in focal lengths. Since the camera is almost entirely depended on for the original record, except only a small amount of ground control, it must be an instrument of precision.*

One of the most important requirements is a lens free of distortion. This is not a common article of commerce, and such lenses were obtained in desired number and variety only after the extended process of search, test, and selection. With the lens used on the Green River survey it was necessary for exact correction and contouring of the photographs to make varying allowance for lens distortion. Subsequently, a series of lenses have been put in use so free of distortion that it can be found only by the most exacting optical tests. Thus, it ceases to be a factor affecting the accuracy of the mapping. In other words, the resulting image displacement cannot possibly be plotted and is so microscopic that it can be ignored in elevation determinations from stereoscopic photographs involving measurements down to 0.001 in. or less on the stereometer. The most satisfying improvement, however, has been made in the instruments used for such measurement and reprojection of the photographs to eliminate the effects of camera tilt. The newest instruments for this purpose have shown the most gratifying results in further labor-saving in the office processes, in minimizing the required ground control,

* *The Military Engineer*, Vol. XVII, No. 96, November-December, 1925.

even extending the mapping for considerable distances without additional control, and in producing accurate results in comparison with the most exacting ground check of completed maps.

In attempting to explain such operations and instruments in readable form and reasonable limits, the writer is conscious of the difficulty suggested by Mr. Faison. For the engineer schooled in the old methods of mapping and unfamiliar with photography, analogies are useful, beginning with the simple one between the camera and the human eye. The pupil of the eye is a lens and the lid is analogous to a camera shutter. As the iris of the eye affects the amount of light admitted to the pupil, so the camera has an iris diaphragm by which the lens may be "stopped down". The retina of the eye corresponds to the focal plane of the camera and each receives its image without the re-erection familiar to the engineer in some of his ground instruments. In the case of aerial photographs taken for mapping purposes with the negative approximately level, such re-erection has no place, but there is the problem of orienting the views, correcting them for slight departure from the horizontal, and overcoming scale changes due to varying elevations of the camera and ground surface. F. E. Weymouth, M. Am. Soc. C. E., has pointed out the analogies and differences between ground and aerial surveys.* It is as if there were a series of plane-table sheets made without exactly leveling the table and using slightly different scales, but with characteristics permitting their correction and conversion to true scale maps in the office. As Mr. Faison states, it is only by actually seeing the office operations, by manipulating the new mapping instruments, that their relative simplicity is appreciated and the new method clearly understood.

Mr. Matthes† offers the best evidence of this by being confused with European processes and instruments quite dissimilar. In comparison, the new method minimizes the required ground work and simplifies its character. It is being applied to small scale mapping. Work at a scale of 1: 62 500 is under way for one Government, similar demonstrations begun for two others, and the expressed desire of numerous others awaits only the accumulation of sufficient instruments in excess of current requirements for commercial work on larger scales.

Much of Mr. Matthes' discussion of the new method is best illustrated by his concrete example of parallax value for which he arrives at a figure of 0.00043 in. for a 10-ft. contour. Actually, under his assumptions of photographic base, lens height, and scale, Equation (1)‡ indicates a parallax measurement of 0.006 + in., or about 1 350% greater than his figure. Photographic enlargement in the office processes would make the measurement still larger, and photographs taken as he suggests, if they were precise records on flat glass plates, would readily permit accurate location of contours with a 5-ft. interval. With equal or greater facility 10-ft. contours could be located on such views, but this could readily be done with far fewer photographs for

* *Engineering*, Vol. CXXI, No. 3138, February 19, 1926.
† *Proceedings*, Am. Soc. C. E., December, 1926, Papers and Discussions, p. 1989.
‡ *Loc. cit.*, March, 1926, Papers and Discussions, p. 374.

a given area than his assumptions would involve, with corresponding economy in various steps of the process.

There is a demand for mosaics of air photographs, which are useful for land exploitation and numerous other purposes. Some of these purposes are of an engineering nature; the writer has cited the case of a preliminary investigation for power development. Mosaics may be especially useful in connection with transmission line location and the acquisition of related right of way. Where the population is relatively dense, property value high, and the right of way by far the heaviest item of cost, such photographic products of aerial surveys are almost invaluable. Interests with which the writer was connected had occasion to locate a high-power transmission line through about fifty miles of such country. The area is a network of improved highways, railroads, and other rights of way, connecting numerous towns, and the countryside abounds in highly cultivated farms. Within a zone or strip of this country the choice of final location was greatly influenced, if not entirely dominated, by the expense for right of way. Flying about 10 000 ft. above the ground an aeroplane traversed this zone and a series of overlapping "vertical" photographs was taken of the strip. The pictures included a few known ground distances, and using these and the precise office instruments already described, a graphic triangulation served to orient the photographs and approximately bring them to common scale for a datum plane. On mosaics of such pictures a line was laid down that would avoid the most expensive right of way and minimize crossings of a nature requiring special construction under the law. The county records served to identify the property lines on the photographs and made it possible to list the owners of rights to be acquired.

All this could be done without premature disclosure of plans. When the right-of-way man was ready, he proceeded unhandicapped by the publicity attending survey and location on the ground. The ground survey was limited to the final exact location, following the acquisition of rights. Here, too, the aerial survey may aid. The photographic plates are kept on file, available for determination of relief as described, and in any event copies of the photographs and mosaics remain in the office, continuously useful for reference on account of their vivid showing of detail. Coupled with the records or testimony of the aerial photographer as to the time of taking, they may serve later as evidence in settling certain kinds of disputes, evidence of a nature most compelling and understandable to a jury. Thus, from the stage of preliminary examination to the final day in Court, the aerial survey may function.

The writer is, therefore, well aware of such utility and the demand for mosaics, but believes in looking all the facts in the face. The mosaic should be called a mosaic and not a map. It is distinctly superior to any conventional map in vivid nature and wealth of detail, but it supplies no elevations and is distinctly inferior to a good map with respect to scale, as those who have been tempted to rely on it beyond its powers have found to their sorrow. This statement needs one modification. If the ground is level and the photographs

free of tilt a mosaic may closely approach a true plan map, but as the ground usually tilts and aerial photographs nearly always do, such modification is in the nature of an exception to prove the rule. Doubtless this is why the U. S. Board of Surveys and Maps refuses to call a mosaic a map, following the action of its Photographic Committee of which Mr. Matthes is a member.

As Mr. Matthes suggests, the engineer may find mosaics of most use to him through their value as a vehicle in conveying information to his client. In their hurry a board of directors or a busy executive will probably get a broader conception of a project from well explained pictures than from maps, profiles, and detailed drawings, but the engineer will probably not abandon the use of the latter. The writer confesses that he is thinking of maps from the engineer's viewpoint. He cannot agree that elevation contours are losing ground or share Mr. Matthes' rather gloomy view of placing little trust in the scaleability of any kind of map, although that is the safe rule to apply to mosaics.

Mr. Matthes disagrees with Major Fiske as to the completeness of the description of the new mapping process, and as to this the writer agrees with Mr. Matthes. The process had been previously described only to the extent possible within the limits of a brief magazine article.* The writer's attempt was to set forth clearly some of the essentials from the beginning. Thus, his effort goes further as a discussion of the problem than a detailed account of its solution. It is to be hoped that it will be followed, as it doubtless will, by others describing more fully the new process of topographic mapping, the precision instruments evolved, and the improvements in both that are being accomplished.

Nearly a decade of painstaking experiment preceded the present commercial application.† One instrument after another was designed, built, put to practical test, and succeeded by something nearer the goal. The persistence, ingenuity, and unfailing support of an ideal constitute an incident worthy of record in the history of American science. Sometimes it is said that in the hurry of the present tremendous industrial progress, technical research in this country lags in comparison to the applied science of engineering. It is a pleasure to record one of many instances to the contrary, to know that although this development has reached the point where it is possible to give it a place in the practical field and literature of engineering, the research and experiment are continuing and further advances materializing.

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* *Engineering News-Record*, Vol. 94, No. 15, April 9, 1925.

† "Research Narrative," Vol. 5, No. 7, Engineering Foundation, New York, N. Y.

FIELD PROCEDURE OF ADJUSTING THE GREAT CIRCLE LINE TO THE RHUMB LINE

Discussion*

By N. B. Sweitzer, Assoc. M. Am. Soc. C. E.†

N. B. Sweitzer,‡ Assoc. M. Am. Soc. C. E. (by letter).§—In his discussion Mr. Hodgson‖ brings up the question of control on State and National boundaries and emphasizes the fact that these isolated and unconnected astronomical determinations are not actually on the parallel or meridian intended and as described in the accompanying field notes and calculations. This fact has been well known to many field engineers who have retraced old lines or who have had to tie into these boundaries. What has often appeared as a paradox is the fact that a line astronomically determined with great expense and by undisputed experts in their professions and with the best and most expensive equipment would fall far short of accuracy as regards azimuth and, only too frequently, measurement, when compared with the ordinarily determined azimuths and measurements of the average field engineer, using modern but commercial instruments.

The writer's attention was first called to this fact nearly thirty years ago on the boundary between Canada and the United States, north of Roseau Lake in Minnesota. The boundary was run through a cedar swamp, and local surveys were closed on the tangent line, cut as a skyline, and not on the actual parallel, which was not marked except by posts set far apart,. This gave a ragged traverse and not the actual parallel as described in the field notes. The limit of error of azimuth on most of these major lines, astronomically determined, is given as 2″ or 3″ of arc, but the fact is that the average transit instrument, with 1′ of arc as the least count of the verniers, easily picks up errors of many minutes on the actual lines as fixed on the ground. Mr. Hodgson mentions many specific instances of this condition, scattered over a large area. The writer can add to this list materially but, as his observations have been confined to errors of azimuth and measurement, he has not been in a position to determine the total error.

Isolated determinations of latitude and longitude are beset with many sources of error. Some can be guarded against and eliminated, but not all. Even after one of these isolated positions has been successfully and accurately

* Discussion on the paper by N. B. Sweitzer, Assoc. M. Am. Soc. C. E., continued from October, 1926, *Proceedings*.

† Author's closure.

‡ Dist. Cadastral Engr., U. S. Interior Dept., Gen. Land Office, Denver, Colo.

§ Received by the Secretary, November 18, 1926.

‖ *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1699.

determined, it is not constant. Latitude and longitude change; they are not permanent institutions. The experts who have been engaged in this work, especially in the past, were thoroughly conscientious, but their enthusiasm for their specialty undoubtedly caused them to overlook certain weaknesses in their methods. Too much emphasis was given to complicated theory and volumes of abstruse calculations with hair-splitting, overlapping niceties which were confusing instead of enlightening and made the authors the sole interpreters of the final results. This condition placed the notes or records in an unassailable position and, when the actual corner positions were legally accepted, there was really nothing to be done about it.

As Mr. Hodgson has pointed out, one of the many errors that beset these isolated astronomical determinations is the deflection of the vertical. This has been pointed out by many authorities on the subject. The late John F. Hayford, M. Am. Soc. C. E., emphasized this in his admirable treatise on "Geodetic Astronomy", but evidently this and other sources of error have been disregarded. The only way to eliminate this deflection is to make adjustments from other accurately determined positions. The answer to all this is that a practical and comprehensive system of triangulation should be an adjunct of every astronomical determination for latitude and longitude; they belong together and are incomplete, unless united.

With the introduction of such modern appliances as radio there is a practical side to astronomical determinations for longitude. This can be taken on almost any condition of terrain. Triangulation is difficult and expensive under some conditions, as in the heavily wooded rolling country west of the Cascade Mountains, in Washington and Oregon and in, say, the Valley of the Amazon. Lengthy, intermediate points will have to be filled in by closed primary traverses, especially in heavily wooded and difficult country. Where such valuable property as mineral and oil lands are at stake, these traverses should be reduced to the loxodrome and it is here that the writer's method could be profitably used. A triangulation can only touch occasional points of an objective boundary; where instrumental connections to the line are used, this system can be abandoned and other methods adopted, such as the back and fore-sight transit line, direct measurement, etc. All these methods introduce their various sources of error, not found in the triangulation. Manifestly, the calculations for these secondary lines do not have to be carried out to the extreme nicety of those in the triangulation network and, even if they were, it would be useless, as, instrumentally, they could not be executed on the ground as calculated. The writer, therefore, thinks that the problem of adjusting the great circle to the rhumb line, as suggested and detailed in this discussion, is amply accurate for any line actually run on the ground and tied into properly adjusted triangulation stations. He heartily agrees with Mr. Hodgson in his discussion.

Mr. Faris* suggests the relation of the rhumb line to the circle from the parallel to the meridian and notes that it will not be a plane section of the ellipsoid except at these points. He, however, states that for the distances

* *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1704.

specified in the paper the approximation of the circle is sufficient to answer all practical purposes, and the field procedure is justified. This element in the problem as to what reception the principle would receive from Government surveying authorities, caused the writer more or less worry. He compared the differences but could find nothing except extremely complicated formulas by which to make comparisons; these were entirely too unwieldy for field purposes and the differences were so small for the usual latitudes that, as far as field work was concerned, they did not exist. Mr. Faris' well-known reputation as a specialist in this particular field confirms the findings. The tendency of such men is extreme conservatism, especially on these subjects, and it is gratifying that he has taken the time to solve this problem. He is to be thanked for his contribution to this subject by the introduction of figures, formulas, and tables. The table of offsets (Table 3*) is especially valuable, carried out as they are to smaller decimal places.

Mr. Corti's† method of locating a loxodrome by a series of chords is quite interesting, and his mathematical handling of the subject will appeal to many, especially those interested in the problem usually stated in the terms of text-books on the subject. He has simplified the problem without sacrificing accuracy that could be identified in the field work and has suggested some valuable points in the practical field location; namely, the possible verification of the work by a process of checking the chord against the tangents. This amounts not only to double chaining but checks calculated offsets against each other from tangents and chords. Mr. Corti has used 5′ and 10′ as differences of longitude in his example and this corresponds closely with that used in the example suggested in the paper. This is amply close, however, to compare the different methods and prove the results.

In a letter dated July 16, 1926, Dr. O. S. Adams, Mathematician, U. S. Coast and Geodetic Survey, called attention to and suggested that the primes be omitted from the *B*, *C*, and *D* factors in the formula‡ quoted from Smith's "Surveying", stating that these factors are taken out for the latitudes of the known point and not for the unknown point. As this book is used as a textbook in many colleges, this is important and has undoubtedly caused confusion in presenting this problem.

The writer is greatly indebted to Dr. Adams and wishes to thank him for his trouble in going over this problem. His mathematical ability is so well known in the profession as to give added value to his opinion that the writer's solution is "sufficiently accurate for all practical purposes".

Mr. Hosmer§ has brought out some valuable points and, as he also is a recognized authority, his statements will bear weight. Simplicity, lucidity, and accuracy are what the field engineer demands, and the elimination of unnecessary complications.

Stripped of unnecessary detail, the development of the method was as follows: The writer first found what angle at the pole would subtend 1 mile

* *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1706.
† *Loc. cit.*, p. 1709.
‡ *Loc. cit.*, May, 1926, Papers and Discussions, pp. 880–881.
§ *Loc. cit.*, October, 1926, Papers and Discussions, p. 1707.

at the equator, using Clark's value (1866) of the earth's flattening. This angle was found to be 0′.867403. In terms of a railroad curve, this angle was substituted for the angle at the center; but the angle at the center equals the intersection angle and this, in the present case, will also equal the convergency, which becomes 0′.867403 tan ϕ. The 1-mile unit was found to be impracticable for field use, as back-sights become confused by running this traverse, and deflection angles become too numerous and small—less than the least count of the verniers—and also are apt to come in draws and in places where it was impossible to set up the instrument. Therefore, the 6-mile tangent, was adopted as a unit and the equation became, sin θ (5′.20442 tan ϕ) = intersection angle, 6-mile tangent, which, for field use, can be written in the form, sin θ (5′.2 tan ϕ), and,

$$d\lambda' = \sin\theta \left(5'.2 \times \frac{D}{6} \text{ sec. } \phi_m\right) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$-d\phi' = \cos\theta \left(5'.2 \times \frac{D}{6}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$-dZ' = d\lambda \sin\phi_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equation (2) gives the difference of longitude; Equation (3), the difference of latitude; and Equation (4), the difference of azimuth. Closer results can be secured by using,

$$d\phi' = D \cos\theta\ (0'.8733 - 0.0001 \times \phi_m).$$

If closer results are required for office use, the intersection angle for the 6-mile tangent for the loxodrome, using Clark's values for the spheroid, can be secured by the following formula:

$$\sin\theta\ (\tan\phi_m \times 5'.20442 \pm d\cos\theta)$$

using the following tabulation for $\pm d$ and interpolating for intermediate values:

Latitude.	$\pm d$.	Latitude.	$\pm d$.
0°	+ 0.0355	54° 45′	0.0000
9°	+ 0.0342	63°	— 0.0067
18°	+ 0.0304	72°	— 0.0125
27°	+ 0.0246	80°	— 0.0160
36°	+ 0.0171	90°	— 0.0176.
45°	+ 0.0089		

For a 1-mile tangent, this will become:

$$\sin\theta \left(\tan\phi_m\, 0'.8674 \pm \frac{d}{6}\cos\theta\right)$$

The results obtained by using the equation for the sphere and applying it to the example under discussion makes the intersection angle for the 6-mile tangent, loxodrome N. 47° E., 3′.5494. The general equation for the spheroid, suggested in this case, that is,

$$\sin\theta\ (\tan\phi_m \times 5'.20442 \pm d\cos\theta)$$

makes this same angle, 3′.5544.

The circular curve on the surface of a sphere and spheroid in terms of the functions affecting a circular curve on a plane surface by methods in use by railroad engineers has been considered herein. It has been only in more recent years that simple curves have given way to more involved spirals to ease off the angles at the point of curve and point of tangent. Different methods were used, namely, lengthening the radius at the point of curve and point of tangent; making a compound curve; or, more generally, by use of the transition curve. These curves, generally speaking, were of a radius of only a few thousand feet and affected the central, or intersection, angle by only a few seconds (a minute or more at most). When these short radii are compared with the radii of the curves under discussion, thousands of miles long, it may be readily seen that the effect of change in the intersection angle by using the equation for a loxodrome instead of the circle will be negligible, especially in the latitudes of the United States and Southern Canada. Farther north, to about Latitude 70° to 75°, by using the 1-mile tangent, they will approach so close as to practically coincide. An illustration of this justifiable substitution of an approximation, which may be said practically to equal a rigid determination, is in the equation for determining the tables in general use for inclined sights in stadia work. These tables are based on the equation,

$$K\,r\cos^2 V\,(c+f)\cos V$$

in which,

r = vertical reading;
V = observed vertical angle;
K = wire interval, or ratio;
c = distance from center of instrument to object-glass;
f = distance from plane of cross-wire to object-glass.

This is under the assumption that the sight along a "vertical angle" at the inclined rod makes a right angle with the rod held vertical. This is not true, but the angle is so nearly a right angle that it does not affect the practical results and the equation has been adopted as "rigid" by the profession, has become standardized and rightly so, and is correct. The writer is in complete accord with Mr. Hosmer when he suggests that the circular arcs give practically the same line on the ground as the loxodrome.

Mr. Hughes' discussion* is interesting, particularly in his description of the evolution of the rhumb line. He brings together the Middle Western farmer with his fence posts and the deep-sea sailor with his mariner's compass on a wheelbarrow, and so illustrates the mathematical principles involved that "he who runs may read." He confirms the correctness and practicability of the formulas used in the paper. The writer is pleased that Mr. Hughes has joined in the discussion, especially on account of his extensive experience in the practical solutions involved.

In all restoration of old and obliterated lines where such lines are boundaries of deeded lands or accepted political subdivisions, etc., certain fundamental rules have been laid down by the Courts for the guidance of surveyors and engineers. Justice Cooley, of the Supreme Court of Michigan,

* *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, p. 1701.

has written some valuable opinions and contributed considerably to the subject. Since those opinions were expressed many important cases have been decided by all the Courts, especially the Supreme Court, all along the same lines. Among the general rules are that fixed monuments hold over course and distance and that a properly accepted and confirmed surveyed boundary is unchangeable. There are many fundamental rules of the same tenor. In restoring a lost boundary engineers should follow in the footsteps of the original surveyor. To do this they have his field notes and plat and, possibly, if the survey is not too old, living witnesses as to what he did and where he placed his monuments. This is all along the line of evidence and competent testimony. When this is lacking or incompetent and contradictory, then the skill of the engineer or surveyor has to be exercised, but, following proper legal procedure and accepted mathematical and technical practice. There are always errors to harmonize, and in harmonizing these errors the skill of the engineer is shown. They are not to be crudely guessed at, but to be scientifically treated, both mathematically and legally. In regard to the original California surveys mentioned by Mr. Hughes, apparently these are mostly in two classes, Spanish grants and cadastral surveys made by the Government. Now, as the case under discussion relates to transit or great circle lines and the loxodrome or solar and needle lines, it is better not to take up extrinsic matters.

The Government survey was based on the loxodromic curve, not only mathematically in fact and by execution, but by legal enactment and judicial decision. First, the major lines—standard parallels—follow the parallel of latitude, and guide meridians follow the proper meridian; then come the ordinary correction lines and check of township, town, and range lines, all following this same system. In the subdivision of the unit township, the section lines correspond to this general system. In the effort to make each section 1 mile square, the loxodromic condition is recognized by running adjacent north and south section lines with a bearing corresponding to the latitude of the township. To confirm all this, the field notes, maps, and returns of all lines, major and minor, give the courses and bearings as a loxodromic curve. These surveys are throughout loxodromic, *de facto* and *de jure*.

Now, as regards the larger tracts known as grants, it will be found that both the great circle and the rhumb line are used. This will be shown first by referring to the record or wording of the grant and then by the surveyor's notes or the method used in running out these boundaries. The wording of some boundaries will call for a line from some peak to another, a great circle, along some river or water-shed, being metes and bounds; and then it will revert to the loxodrome, with course and distance. In retracing or restoring these grant lines, these conditions will have to be taken into consideration. The method used for restoration must correspond to the wording of the original record of the grant, the treaty, proclamation, or instrument creating it, and also to the methods used by the engineer or surveyor whose final survey has been properly accepted.

In Mr. Hughes' examples,* which are hypothetical, so many unknown conditions may exist respecting transfers, deeds, methods of the original survey, and the question of the authority by which the survey was made, that it is difficult to give an answer. The example as to the method of running a line in Latitude 43° N. a course of N. 47° E., 6 miles, would depend upon what was intended in the wording of the deed or grant and what the original surveyor did. A description of the line would help materially. Was it an air-line from peak to peak of a mountain, or through dense timber and brush? One condition would predispose a great circle; the other that a rhumb line was used, assuming that the description in the deed was ambiguous.

In regard to Mr. Hughes' query about a straight line and the preference of ranchers for the great circle line, they must remember they are living on a spheroid. This was very well and good before the days of Columbus. When the good padres came to California land was cheap and abundant, but land values have appreciably increased, especially in California, and recent investors are more critical about having their land boundaries restored and re-established.

The second problem suggested by Mr. Hughes resolves itself into two parts, namely, (1) determining the class of lines required, whether great circles or rhumb lines; and (2) determining and adjusting the error of closure. The first has been discussed herein in the first problem suggested, and the second part, namely, the error adjustment, has many angles. The writer is rather partial to the form of adjustment suggested in the discussion of his paper on "Retracement-Resurveys, Court Decisions and Field Procedure", by Leonard S. Smith, M. Am. Soc. C. E.,† for adjustment of traverses. This consists of the adjustment of both azimuth errors and proper proportional chaining. Fig. 4 shows a plane graphic field adjustment, unclosed. Course and distance are taken to only the nearest degrees, minutes, and links. The method, however, is rigid.‡ While the adjustments for error in azimuth and proportional chaining are sound legally and mathematically, local conditions should be studied and the history of the case thoroughly considered.

In these examples of Mr. Hughes, he may have in mind the adjusting of a triangle. If the original work was a primary triangulation executed by scientific and competent authorities the angles, of course, should be adjusted by the method of least squares and the side adjustments applied. However, as the triangle illustrated in the example is small and apparently not connected with a triangulation network, the original work would probably have been a secondary triangulation, or even of less importance; in this case, the angles would have been reduced by taking their mean values. All this is discussed in textbooks on higher surveying. As to the examples suggested by Mr. Hughes the main point to bear in mind is that a reproduction on the ground of the actual lines run by the original surveyor or engineer and the re-establishment of all monuments in their original positions are the prime objects. A great many of these cases suggested have to be passed on by the Courts; and

* *Proceedings*, Am. Soc. C. E., October, 1926, Papers and Discussions, pp. 1703–1704.
† *Transactions*, Am. Soc. C. E., Vol LXXV (1912), p. 429.
‡ "Retracement Resurveys," by N. B. Sweitzer, 1918.

the Courts see through legal eyes and precedents and not so much by mathematical reasoning.

The method given in the paper is suggested primarily for the convenience and aid of the field engineer. The theoretical problems concerning the shape of the earth and the elaborate formulas evolved for securing terrestrial measurements in various latitudes and longitudes have been thoroughly and accurately calculated by the most eminent specialists in this field. These formulas and problems are mostly reduced to the geodetic position and are referred to Clark's spheroid of revolution giving results to 0.001″ of arc.

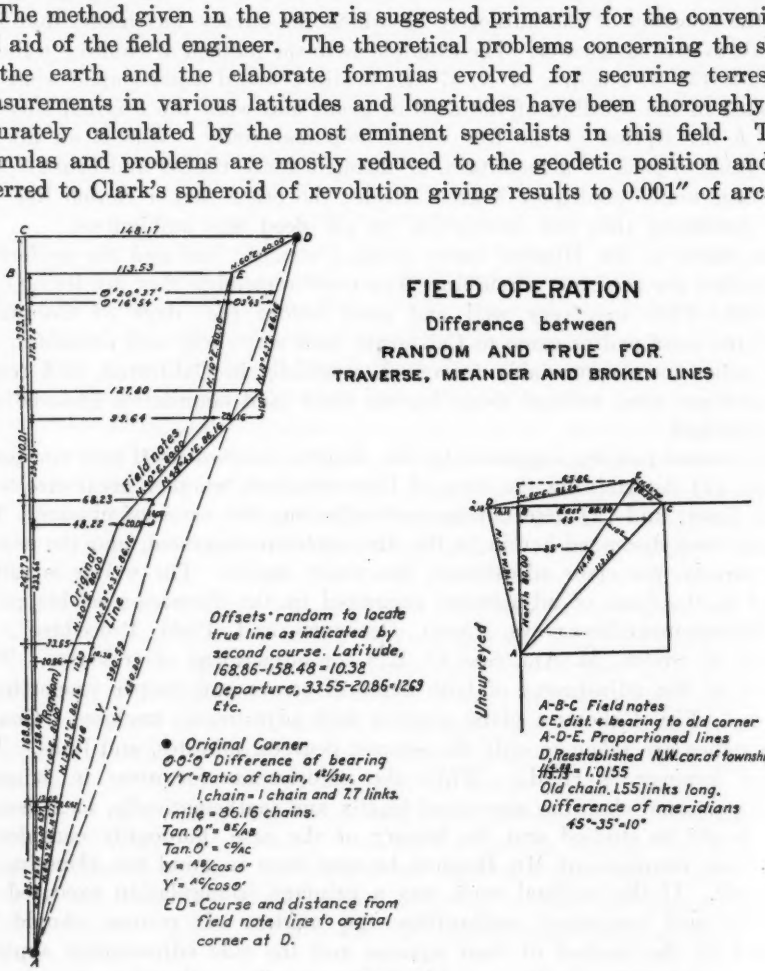
FIELD OPERATION
Difference between
RANDOM AND TRUE FOR
TRAVERSE, MEANDER AND BROKEN LINES
Field notes
Original Line
Offsets random to locate true line as indicated by second course. Latitude, 168.86-158.48 = 10.38 Departure, 33.55-20.86=12.69 Etc.
● Original Corner
0°0'0" Difference of bearing
Y/Y' = Ratio of chain, 42/33, or 1 chain = 1 chain and 27 links.
1 mile = 86.16 chains.
Tan. 0" = B/AB
Tan. 0' = C/AC
Y = AB/cos 0"
Y' = AC/cos 0'
ED = Course and distance from field note line to orginal corner at D.
Unsurveyed
A-B-C Field notes
C.E, dist. & bearing to old corner
A-D-E. Proportioned lines
D, Reestablished N.W. cor. of township
= 1.0155
Old chain. 1.55 links long.
Difference of meridians
45°-35'=10°

FIG. 4.

Nowadays, engineers are engaged mostly in retracements and relocations of lost or obliterated monuments. As original surveys are fast nearing completion it is therefore necessary to restore such monuments, as nearly as possible, in their original positions. This requires taking into consideration the methods used in making the original survey. Practically all these original surveys were made with a needle compass, solar instrument, or simple transit. On major lines (such as State boundaries), sextants, zenith telescopes, and other more refined instruments were used for the latitude and longitude determinations. Practically all these instruments secured their zenith or horizon by the action of gravity, or the plumb line. This determination, however, is

astronomic, not geodetic, and is referred to the actual irregular figure of the earth at that particular instrumental station. This, of course, would include the station error with other errors which, in some instances, would amount to as much as 10″ of arc or more, if reduced to the geodetic position.

It is evident therefore that the more elaborate formulas, including reduction to the geodetic center, would not produce results any closer to the proper final original positions sought than those suggested by the method of the paper. Practically all instrumental field operations incidental to the restoration of lost monuments or original surveys are limited in azimuth readings to the least count of the verniers. Modern practical field experience has generally shown this to be 1′ or 30″ of arc. A more refined instrument than this is too clumsy for rapid and economic transportation and handling. Therefore, why furnish the field engineer with equations and data refined beyond his field execution? In highly refined original work more involved equations are appropriate, especially in obtaining extremely exact results for the final reports and notes. The writer's simpler equations could be used for checks on the final results. If these results varied more than 1″ of arc it would be advisable to check the rigid and more involved methods.

The writer is grateful for the interest taken in this discussion and, especially for the expansion and development of the ideas presented in this method. The unanimity of those taking part, especially from so many widely separated places, and representing different specialties in the profession, is gratifying.

SNOW REMOVAL PROBLEMS OF ORGANIZATION AND OPERATION

Discussion*

By George E. Hamlin, M. Am. Soc. C. E.

George E. Hamlin,‡ M. Am. Soc. C. E. (by letter).§—The value to the motor-vehicle operator and owner, of the free use of an unobstructed highway at all times, may be computed partly in terms of car-mile operation, including interest and depreciation costs. This factor, however, is probably of small account considering the actual loss to the community in delayed delivery of needed items due to non-operation of the highway. The truck in many sections has taken the place of other types of transportation and a return to the original type cannot be made without preparation. Food supplies are not held in quantity in the retail store, dependence for replenishing the depleted stock being placed on the daily delivery by truck from the central warehouse.

Any business organization considers the minimizing of necessary losses as well as the making of profits. This is as true of highway operation as of any other business organization. Capital invested cannot economically be allowed to lie idle when a return can be furnished to the advantage of both the producer and the consumer.

The State Highway System of Connecticut, including bridges and ferries, had an estimated value in 1926 of more than $38 000 000. If interest at 4% only is considered, the daily charge will amount to $4 165, this being a total loss through non-operation of the highway system. The fees received from the gasoline tax, which in Connecticut are devoted in their entirety to maintenance and reconstruction of highways, are also lost. These fees during the fiscal year ending July 1, 1926, amounted to about $2 500 000 and in the three winter months of December, January, and February, to about $550 000, or about $6 100 per day. These two items of interest and loss of gasoline revenue alone amount to more than $10 000 per day.

The snow removal cost in Connecticut, as elsewhere, varies with weather conditions, but probably can be averaged at about $100 000 per year. This amount can be wholly compensated for by interest and loss of gasoline revenue charges for a 10-day period during the winter months.

An additional benefit from snow removal which is of measurable saving in maintenance costs is the reduction of necessary spring patching of sur-

* Discussion on the paper by George E. Hamlin, M. Am. Soc. C. E., continued from December, 1926, *Proceedings*.

† Author's closure.

‡ Supt. of Repairs, Connecticut State Highway Dept., Hartford, Conn.

§ Received by the Secretary, February 2, 1927.

faces. In this one item alone, the State of Connecticut has probably justified the entire cost of snow removal. Where snow is permitted to remain, ruts are formed in which practically all the travel operates. Each vehicle is necessarily equipped with chains, as icy conditions are continuously met. The travel is concentrated in narrow widths in which the chains cut and wear through the road surface for an appreciable depth. Water enters the roadbed through the broken surface and disintegration increases proportionately. This condition results in the necessity for continuous surface patching in the early spring.

Where the snow is removed, travel utilizes the entire width of the highway, the wear is more evenly distributed, and chains are the exception rather than the rule. The ordinary yearly bituminous application to the surface will place the roadway in good condition for summer travel, and patching—always the expensive treatment—is reduced to a minimum.

Maintenance can possibly be defined as the labor and material required to keep all highways at all times of the year in an operating condition commensurate with the just demands of travel.

The writer agrees with Mr. Means* that snow removal costs should be a part of the annual maintenance budget.

Mr. Sevison† has commented on the importance of proper location in construction so that the necessity of snow removal may be reduced to a minimum. In Connecticut, for the past three years, this has been one of the duties of the locating engineer and is definitely considered in determining the economic location of a new or relocated highway. When necessary, cuts are widened to reduce snow hazards and to permit the clearing to the sides of the roadway by plows, rather than to require removal by expensive hand methods.

* *Proceedings*, Am. Soc. C. E., September, 1926, Papers and Discussions, p. 1515.
† *Loc. cit.*, p. 1518.

STRAIGHT LINE PLOTTING OF SKEW FREQUENCY DATA

Discussion*

By Messrs. E. P. Goodrich, Arne Fisher, and Allen Hazen.

E. P. Goodrich,† M. Am. Soc. C. E.‡—All engineers are familiar with the fact that observations with reference to any quantity, whether it be the measurement of a single distance, or data with reference to rainfall or run-off, or the petals of a flower, or of other biological, sociological, and engineering factors, when plotted side by side, will form a series of points which have an ogee shape. Such a curve may be called a distribution curve.

If the items are gathered together, so that the number of each size between progressive units of measurement are grouped, the result is a curve which may be called a frequency curve. A number of general formulas have been found to fit those two curves more or less accurately, depending on the character of the observations, the conditions which underlie the variation in the quantity itself, and other features.

The frequency of occurrence of engineering, sociological, and biological phenomena may, but generally does not, correspond with the frequency at which deviations have been found to occur in the repeated measurement of a fixed physical quantity. The curve which represents the frequency of occurrence of given deviations from the arithmetical mean of such measurements, shown either in quantity units or in percentages of the average, has been found by experience to agree closely with the mathematical curve of which the demonstration was first published by Adrian in 1808 at Philadelphia, Pa., in *The Analyst*,§ generally known as Gauss' "Error Law". Gauss' derivation was published in 1809,‖ but he is said to have stated that he had used it since 1794. In 1837 Hagen gave another method of analysis which has been generalized by Eggington.¶

Fisher** states that "* * * among the leading mathematicians of the present time there exists no uniform opinion as to the truth of the axioms underlying the theory of probabilities."

Note.—The discussion by Mr. E. P. Goodrich is largely subject matter of a talk given by him in presenting this paper for R. D. Goodrich, M. Am. Soc. C. E., at the meeting of the Society on September 8, 1926. Much of the material was provided by Mr. R. D. Goodrich covering results of his studies subsequent to the submission of the paper.

* Discussion on the paper by R. D. Goodrich, M. Am. Soc. C. E., continued from February, 1927, *Proceedings*.

† Cons. Engr., New York, N. Y.

‡ Received by the Secretary, December 21, 1926.

§ See p. 209, "Practical Least Squares," Leland, McGraw-Hill, 1921.

‖ *Loc. cit.*, p. 201.

¶ See p. 15, "The Combination of Observations," Brunt, Cambridge Press, 1917.

** See p. 127, "The Mathematical Theory of Probabilities," Fisher.

Brunt states that:*

"The proofs of the Normal Error Law * * * are based on certain definite hypotheses concerning the nature of accidental errors. It has been shown that, if accidental errors to which a series of observations is liable satisfy these hypotheses, the errors of observation will be distributed according to the normal law. The final justification of the use of Gauss's Error Curve rests upon the fact that it works well in practice, and yields curves which in very many cases agree very closely with the observed frequency curves. The normal law is to be regarded as *proved by experiment,* and *explained* by Hagen's hypothesis. When the curve of frequency of the actual errors is not of the form of the normal curve, we may safely conclude that the nature of the accidental errors concerned is not in accordance with Hagen's hypothesis."

The normal curve has applications in a region where deviations from a mean value are considered, although these deviations are not, properly speaking, of the same nature as accidental errors of observation; for example, it is frequently applied in biological questions. The use of the normal curve in such cases is justified only when the differences between individual cases are produced by causes the mode of action of which is in accordance with Hagen's hypothesis.

In other words, any one can propose a series of formulas with their corresponding curves; and if they fit the facts better than any other formulas, the one which fits best and is easiest of application is probably the most desirable one to use. That is the only reason why the author suggests his formula, because after testing it in a great many cases, he finds with reference to hydraulic data, for example, that it works better than that of Pearson, which has been the formula most used by engineers thus far. In biological data it fits just as well as Kapteyn's or Pearson's formula in all the cases the author has been able to discover.

The test of "goodness of fit" is made by plotting the curves by the two methods, computing the differences, finding their variation, and measuring it by the usual statistical method of computing the standard variation, or other characteristic measure.

There is a further reason why the author suggests the use of his curve—because it is almost exclusively graphical in its detail of application. The simple plotting of the data on the type of skew frequency paper that he has devised makes it possible for any engineer to apply the methods described, at least after a little practice. Furthermore, as stated by a number of those who recently have discussed the other papers which have been written on the subject of frequency and probabilities, it is difficult except in special cases to integrate the frequency curve when the equation is given in the Pearson form. In the author's formula the mathematical relationship between the two curves is obvious and the investigator can pass directly from one to the other without going through the detail of an integration or differentiation. In other words, when one is accustomed to the use of the two formulas he can write them out immediately as soon as the data are secured.

* See p. 17, "The Combination of Observations," Brunt, Cambridge Press, 1917.

The constants which are found in the general form of the curve appear to have specific meanings, which is not the case with reference to any other curves except possibly those of Kapteyn. That is, A is the limit in one direction; B is the limit in the other direction; C, or C divided by D, measures the skewness of the vurve; and the factor, H, depends on the system of logarithms used, the exponent varying with the unit of measurement of the data, whether it be "feet" or "meters", "inches per acre", or other unit.

Another reason why the author's formula and method are more adaptable than the others is that it is usable with relatively fewer observations. If one undertakes to compute the different factors involved in Pearson's equation, for example, he discovers that a large number of observations is necessary before anything like real accuracy can be obtained. In fact, the distribution curve, the frequency curve, the first moment curve, and the second moment curve must either be computed or actually graphed and measured, in order to discover the four constants which Pearson usually employs.

In the author's formula the whole method operates in the other direction. The investigator plots his data and from that computes the factors or constants directly. Having determined the relative percentage of time or number of observations that correspond with each proportional group he can work directly on the special paper devised by the author. The first plotting will take some one of a series of typical shapes. Of course, it will be exceptional that the points will follow precisely along any one of the type lines, but the several groups which are indicated in Fig. 24 are typical of the types of data which may be expected.

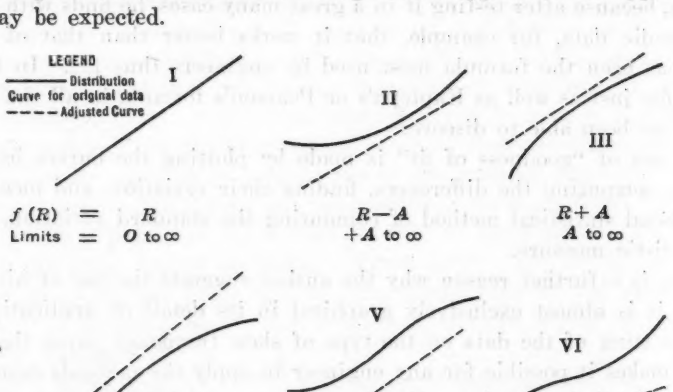

FIG. 24.—TYPES OF DISTRIBUTION CURVES AND CORRESPONDING FUNCTIONS OF THE VARIABLE FOR GOODRICH FORMULA.

The author discovered that, for each type of curve, after the data are adjusted by use of proper constants with appropriate limits as to the observations, the adapted function will finally plot as a straight line. In Type I the original data plot as a straight line and the frequency curve is very close

to the normal curve of error, as is seen in Fig. 1.* Again, in Type II, exemplified in Fig. 2,† it is to be noted that as the curve extends toward the left it becomes flatter and flatter; and if it were indefinitely extended it is easily conceivable that it would become horizontal at about 8 on the vertical scale; in other words, 8 is the limit to be set in the value of A. It is also to be noted in the equation given in Fig. 2 that $(R - 8)$ is the actual quantity used. In other words, the plotting of the curve often determines the actual limits of the data. This simply means that to a certain point—a distance, A, on the measurement—there is no precipitation each year, or a certain minimum number of petals of a flower, or a certain number of deaths per annum; and after the measurement, A, is passed then the variation is approximately in accordance with the normal law of frequency.

In the next curve type (III), instead of being a subtractive quantity, the limit is above the data, and the difference between the observed figures and the limit is the quantity that corresponds with this law of frequency.

The limits for the fourth form are much more difficult to determine from observation of the plotted data. Few curves of this type are discovered.

When it comes to Type V it is seen from Fig. 8‡ that as the curve extends toward the left it approaches 70 as its limit, and as it extends toward the right it is approaching 150 as its limit. In other words, if one started by drawing a curve which would fairly correspond with the observations, the limits within which this variation occurs would be indicated; and the formula evolved in this particular instance gives $(R - 70)$ as the numerator and $(150 - R)$ as the denominator.

When it comes to the final type (VI), which is very rare, it is usually necessary to select points which appear to represent the curvature fairly well, and solve some relatively simple equations to obtain the limits.

After having tentatively determined the apparent limits, it is sometimes found necessary to make a second or third approximation. Having secured an adjusted line from the proper selection of the limits, the value of C, which is the exponent of the function, is the tangent of the angle between such straight line and the vertical on its under side (Fig. 25). This is true only when the data are plotted on the author's skew frequency paper, because otherwise the scales are apt to be inappropriate.

In all the former cases by measuring the angle the value of C is immediately obtained. That appears, also, from an analysis of the equation itself. The value of H (which depends on the units employed in the data), is often easily found by reading the value of "percentage of time" at the point where the adjusted straight line crosses the horizontal line with unity as its ordinate (Fig. 25) and then applying the equation,

$$h = \left[-\log\left(1 - \frac{t}{100}\right)\right]_{f(R)=1}$$

The factor, H, or K, depends on the system of logarithms used.

* *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1076.
† *Loc. cit.*, p. 1079.
‡ *Loc. cit.*, p. 1085.

The author has devised graphical methods for accurately fixing the limits and, in some instances, has devised various mathematical formulas for the quicker determination of the various items; but normally it is found that the method of trial and error will give quickest results.

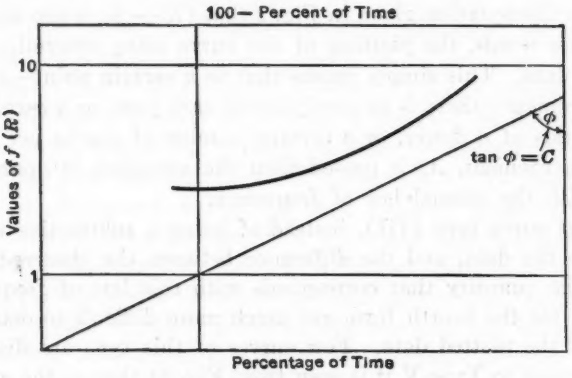

FIG. 25.

Some of those who are already expert in the use of the "normal curve" formulas and who know that in the Pearson, Charlier, and Kapteyn systems the equation of the normal curve is a special case of the general formula, may object that the Goodrich formulas can not be reduced to the same special (Gaussian) form by the substitution of any possible group of constants. This is true, and is because the author has found it advantageous to introduce the independent variable both as a main factor as well as a factor in the exponent of e. This introduction also precludes the possibility of mathematical symmetry in the probability function. It is interesting to discover, however, that it is possible to select the constants so as to make a probability and its integral or "duration", curve that are substantially equivalent to the normal curve and its integral curve. This is shown in Fig. 26 and in Table 22.

The integrals of the normal error curve will plot nearly as a straight line on the author's skew probability paper.

Again, one of the fundamental assumptions on which the normal error function is based does not hold for certain types of physical observations; that is, positive and negative errors occur in equal numbers. Clerk Maxwell points out one such instance in observing occultations.*

One great advantage of the Goodrich system of formulas is that it is possible to integrate with ease any numerical probability curve with fractional as well as integral exponents. Certain factors require the use of gamma functions when fractional exponents occur, but gamma functions are no more difficult to learn to use than logarithms.

An incidental item of interest with reference to the author's formulas is that an application of the method often used in connection with the normal curve to show that the arithmetic mean is the most probable value of all those

* See Encyclopædia Britannica, 9th Edition, Vol. XIX, p. 781, footnote.

observed, gives the same result when the author's exponent, c, is made equal to unity, and that the sum of the powers of deviations or differences is in each case a constant, varying from zero, for $c = 1$, to $\frac{1}{K}$, when $c = \infty$.

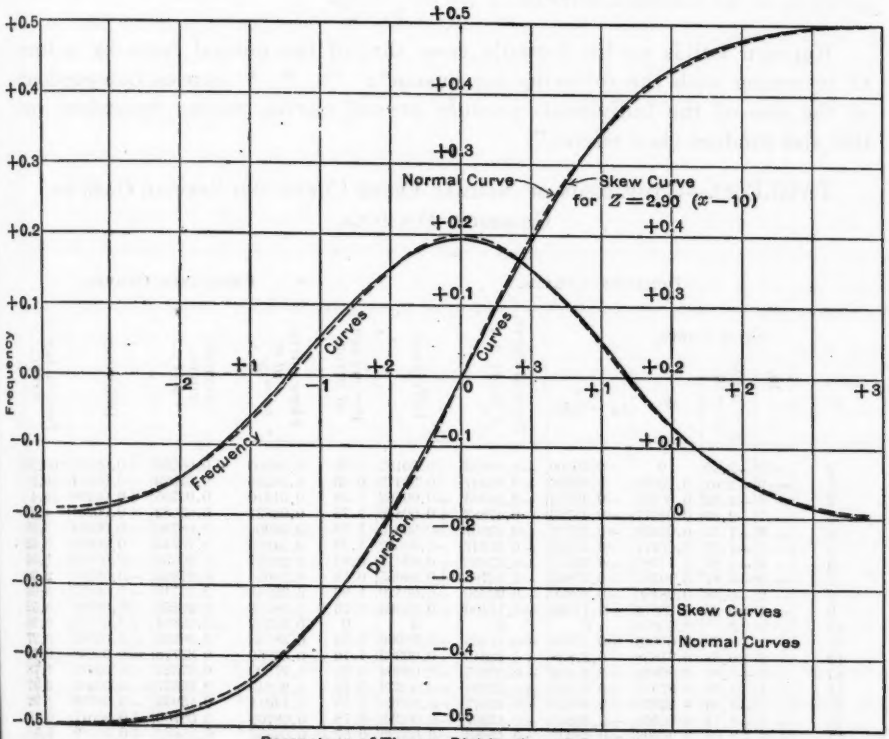

FIG. 26.—COMPARISON BETWEEN NORMAL CURVES OF ERROR AND SKEW CURVES.

The general form of the skew frequency curve equations of Pearson, Charlier, Kapteyn, and Goodrich (using similar nomenclature as far as possible), are as follows:

Pearson:

$$y = e^{\int \frac{(x+a)\,dx}{b_0 + b_1 x + b_2 x^2}}$$

Gram-Charlier:

$$y = A_0\,\phi(x) + A_3 \frac{d^3\,\phi(x)}{dx^3} + A_4 \frac{d^4\,\phi(x)}{dx^4} + \ldots.$$

Kapteyn:

$$y = \frac{1}{\varepsilon\sqrt{2\pi}} \frac{d\,[F(x)]}{dx}\, e^{-\frac{1}{2\varepsilon^2}[F(x) - F(x_0) - M]^2}$$

Goodrich:

$$y = \frac{n}{2\varepsilon^2} \frac{d\,[F(x)]^c}{dx}\, e^{-\frac{1}{2\varepsilon^2}[F(x)]^c}$$

In the Gram-Charlier formula, $\phi(x) = \frac{1}{\varepsilon\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\varepsilon^2}}$, so that his first term is nearly the same as the others; in the Kapteyn formula, $F(x)$ is entirely general; in the Goodrich formula, $F(x) = \frac{x-a}{b-x^a}$.

Kapteyn builds up his formula from that of the normal form by a line of reasoning with the following conclusion*: "* * * causes independent of the size of the individuals produce normal curves, causes dependent on this size produce skew curves."

TABLE 22.—Comparison of Normal Error Curve and Special Case of Goodrich Formula.

Duration Curves.								Frequency Curves.			
Skew Curve.					Normal curve, $\int_0^x \phi(t)\,dt$.	Difference.	Percentage of error.	Skew curve, $w' = \frac{dw}{dz}$.	Normal curve, $\phi(t)$.	Difference.	Percentage of error.
x.	$x - 10$.	$z = 0.290(x - 10)$.	y.	$w = (y - 0.5)$.							
0	−10	−2.90	0	−0.50000	−0.49813	+0.00187	0.38	0.00000	+0.00595	+0.00595	100.00
1	− 9	−2.61	0.00034	−0.49966	−0.49547	+0.00419	0.85	0.00386	0.01323	+0.00937	70.9
2	− 8	−2.32	0.00335	−0.49665	−0.48983	+0.00682	1.39	0.01910	0.02705	+0.00795	29.4
3	− 7	−2.03	0.01277	−0.48723	−0.47883	+0.00840	1.75	0.04831	0.05082	+0.00251	4.93
4	− 6	−1.74	0.03278	−0.46722	−0.45907	+0.00815	1.78	0.09204	0.08780	−0.00324	3.69
5	− 5	−1.45	0.06741	−0.43259	−0.42647	+0.00612	1.44	0.14866	0.13943	−0.00923	6.62
6	− 4	−1.16	0.11985	−0.38015	−0.37698	+0.00317	0.84	0.21387	0.20357	−0.01030	5.06
7	− 3	−0.87	0.19160	−0.30840	−0.30785	+0.00055	0.18	0.28054	0.27324	−0.00730	2.68
8	− 2	−0.58	0.28180	−0.21820	−0.22240	−0.00420	1.89	0.33940	0.33718	−0.00222	0.66
9	− 1	−0.29	0.38674	−0.11326	−0.11409	−0.00083	0.73	0.38052	0.38250	+0.00198	0.52
10	0	0	0.50000	0	0	0	0	0.39582	+0.39894	+0.00312	0.78
11	+ 1	+0.29	0.61343	+0.11343	+0.11409	+0.00066	0.58	0.38148	0.38250	+0.00102	0.27
12	+ 2	+0.58	0.71858	+0.21858	+0.22240	+0.00382	1.72	0.33960	0.33718	−0.00242	0.72
13	+ 3	+0.87	0.80849	+0.30849	+0.30785	−0.00064	0.21	0.27809	0.27324	−0.00485	1.78
14	+ 4	+1.16	0.87970	+0.37970	+0.37698	−0.00272	0.72	0.20840	0.20357	−0.00483	2.37
15	+ 5	+1.45	0.92970	+0.42970	+0.42647	−0.00323	0.76	0.14211	0.13943	−0.00268	1.92
16	+ 6	+1.74	0.96266	+0.46266	+0.45907	−0.00359	0.78	0.08763	0.08780	+0.00017	0.19
17	+ 7	+2.03	0.98203	+0.48203	+0.47883	−0.00320	0.67	0.04853	0.05082	+0.00229	4.5
18	+ 8	+2.32	0.99222	+0.49222	+0.48983	−0.00239	0.49	0.02397	0.02705	+0.00308	11.4
19	+ 9	+2.61	0.99700	+0.49700	+0.49547	−0.00153	0.31	0.01048	0.01323	+0.00275	20.8
20	+10	+2.90	0.99898	+0.49898	+0.49813	−0.00085	0.17	0.00402	+0.00595	+0.00183	30.8
Average.	.....	.....	.....	.....	.....	.....	0.84	.....	.....	.....	10.00*

* This does not include the 100.00% error.

Goodrich has worked out the detailed application of an independently developed general formula of which that of Kapteyn may almost be considered a special case and in general the same basic reasoning will apply.

Kapteyn also remarks*:

"In [some] * * * cases * * * the effect of the causes of deviation cannot be independent of the dimension of the quantities observed. In such cases * * * the frequency curve will be a skew one."

In other words, if $y = f(x)$ has a normal distribution, $y = f(z)$ will have skewed distribution, where, for example, $z = x^2$, and the origin is not at the mode.

* See p. 13, "Skew Frequency Curves in Biology and Statistics," Kapteyn.

For the benefit of the novice in the use of frequency data, it is felt that an amplification of the description of the method given by the author is advantageous. First will be described in some detail the easy means of preparing and arranging data for plotting and, following this, an explanation of the methods of selecting the most suitable form of curve and of interpreting the results of the analysis. A few selected examples will be given, beginning with the simplest and easiest form, which involves only two constants to determine the equation, and advancing step by step to the most general equation with six constants, of which one is determined directly from the data. All these examples were supplied by the author.

For the first illustration consider the data of Table 3,* and Fig. 1, but use the ratio of the total run-off for each year to the mean for the eighteen years of record, instead of the percentage, as given in the table. The advantage of this will be pointed out later. These ratios are first arranged and tabulated in the order of their magnitude beginning with the lowest, as in Column (1), moving the decimal point two places to the left. When there are from about ten to forty or fifty records or observations of practically equal weight, each may be considered as the "class mark" of a series of equal percentage intervals. With n records there will then be n intervals, each covering a range of $\frac{100}{n}$ per cent. Consider the records numbered in order from 1 to n. Then for plotting each record should be assigned the percentage corresponding to the center of the interval of which it is the class mark. For any record, as the rth, the corresponding percentage will be $100\,\frac{2r-1}{2n}$. It is at once evident that the numerators of these fractions will form the series of odd numbers from 1 to $2n - 1$. The values of these fractions can be rapidly calculated on the A and B-scales of an ordinary slide-rule with only one setting of the slide. After calculating the percentages up to 50%, the remaining values are best and most accurately obtained by subtraction, 100 minus the first value giving the last one, etc. These percentages are set down in Column (2) and the data are ready for the first trial plotting on skew frequency paper.

With the data plotted as shown in Fig. 1, it is at once evident that they lie very close to a straight line which can be located by the eye with the aid of a straight-edge or a fine thread. If the points are too irregular for this method, or if greater accuracy is desired, the line may be located by the method of centroids for logarithmic plotting. Since the author's method gives all the points equal weight, the centroid method is particularly applicable.

A great many questions about the run-off of the river at this station can now be answered directly from the curve. For example, one-half the time the total run-off will lie between the values at the intersections of the curve with the 25% and 75% ordinates, that is, between about 0.8 and 1.18 of the mean. The difference (0.38) may be called the range of average run-off or, briefly, the

* *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1077.

"average range". One-half this range is termed by Yule*, the "quartile deviation" or the "semi-interquartile range".

It is evident that one-half the total area under the differential or frequency curve, will lie between these limits. Hence, if the curve were exactly symmetrical and followed the "normal law of error," this quantity would equal the standard deviation. It is therefore an excellent measure of the "dispersion" of the observations and its value can be taken from the curve without the trouble of computing the "root-mean-square" of the differences of the arithmetic mean and each of the observations.

Again, the 50% ordinate intersects the curve at a height of about 0.99 of the mean run-off. Therefore, one-half the total number of observations lie on each side of this point, which is called the "median". With these three readings taken directly from the curve, one can obtain a good idea of the amount and character of the skewness of the curve. Yule gives one definition of "skewness" as the range from the median (50%) to the upper quartile (75%) minus that from the lower quartile (25%) to the median divided by the semi-interquartile range. This may be called the "quartile ratio" to distinguish it from other definitions for the term, "skewness". For this curve, the quartile ratio (Q. R.) is:

$$Q \,.\, R \,.\, = \frac{(1.18 - 0.99) - (0.99 - 0.80)}{\frac{1}{2}(1.18 - 0.80)} = 0.0$$

Hence, this curve is approximately symmetrical, with the mean very nearly equal to the mode. Years of total run-off greater than the mean and years less than the mean are therefore about equally probable, which is an important point in any study of annual run-off for purposes of water supply. The upper end of the curve indicates that a run-off less than 1.6 to 1.7 times the mean may be expected 99 to 99.5% of the time; or to put it the other way, once in from 100 to 200 years the run-off will probably be greater than the figures stated. On the other hand the lower end of the curve shows that values less than from 0.34 down to about 0.28 of the mean are probable once in 100 to 200 years.

This one curve serves to investigate the probability of both unusually high and extremely low run-offs with a minimum of time and labor and with sufficient clearness and certainty to fill the requirements of the average engineer who is seldom interested in frequencies beyond the limits mentioned. The author states that the original purpose of the development of this graphical method of examining skew frequency data, was to make possible a "linear" extension of the "total frequency" or "duration" curve for a reasonable distance beyond the actual range of observations, and this has been done in Fig. 1. This curve is so nearly symmetrical that a similar plotting on Hazen's probability paper would doubtless lead to nearly identical conclusions.

If still further extended below the lower data, the curve reaches to zero while at the upper end it would theoretically extend to infinity, with infin-

* See "Introduction to the Theory of Statistics," by G. Udny Yule, 4th Edition, Charles Griffin & Co., Ltd., Chapter VIII, p. 147.

itesimal frequencies long before it reaches either limit. Some one may object to the infinite limit, but this should not be sufficient cause for the rejection of the entire method in the absence of more numerous and accurate data to give conclusive evidence as to the character of the curve when extended in this direction. Conclusions based on this curve will be as safe as it is possible to obtain within reasonable limits from the data available. While the question of limits will always afford a fruitful and endless subject for discussion, it will be shown later that other of these methods of plotting frequency data provide for a finite upper, as well as lower, limit, whenever the character of the data indicates such limits. In fact, it is impossible to plot the data so as to approximate a straight line in many cases, unless a finite upper limit is used in computing the linear function, which will be discussed later.

In addition to the foregoing questions, it is possible to answer many others directly from the curve. For example, suppose one wishes to know the probability of the occurrence of about 0.8 of the mean, interpreting this as a run-off from 0.75 to 0.85 of the annual average. For the lower of these figures, the curve (Fig. 1) intersects the 20% ordinate and for the upper, the 30% ordinate. Hence, the probability is the difference, or 10%; that is, a run-off of about 0.8 the average will probably occur about once every 10 years. In the theory of probabilities, areas under the differential curve represent probabilities, which therefore can be determined from the difference of corresponding ordinates to the integral curve that has been plotted; hence the great convenience in using the curve instead of the frequency curve.

If the arithmetical magnitude of the "most probable value" of the run-off is desired, it is first necessary to determine the values of the constants in the equation of the curve, which takes the form:

$$t = 100\,(1 - 10^{-h\,R^c}) \qquad (S)$$

in which, R is the ratio of any value of the annual run-off to the average, and t is the corresponding percentage of the total time. Also, the index, c, is the slope of the line drawn through the points with respect to the axis of run-off while the modulus, h, is the intercept. In this case its value is $-\log\left(1 - \frac{t}{100}\right)$ for the percentage of time at the intersection of the curve and the horizontal line at the value of R equal to unity. If the percentages with respect to the mean run-off are used instead of the ratio to the mean, the intercept would have fallen outside the limits of the diagram, as in Fig. 1. The values of the index and modulus are easily taken from the curve and the resulting equation is, therefore:

$$t = 100\,(1 - 10^{-0.3\,R^{3.85}})$$

or,

$$t = 100\,(1 - e^{-0.69\,R^{3.85}})$$

The most probable value of the run-off is now obtained from the following formula for the "mode" which indicates the location of the maximum point

on the frequency curve and the point of inflection on the duration curve when they are plotted with ordinary rectangular co-ordinates:

$$\text{Mode} = \left(\frac{c-1}{ck}\right)^{\frac{1}{c}} = 1.01$$

in which, $k = \frac{h}{M}$ is the modulus of the equation when the base, e, of the natural system, instead of the base of the common system, is used.

The foregoing example illustrates the simplest and easiest conditions to analyze graphically, but often the data will follow a decided curve when plotted as described. Examples of such curves are shown in Figs. 2 to 6 of the paper. While the duration curves in Figs. 2, 3, 4, and 6 are all more or less concave upward, they illustrate widely differing forms of frequency curves. All of them are decidedly unsymmetrical. In Fig. 2* the mode is slightly less than the mean, while in Fig. 3† it is only about two-thirds of the mean, giving very large positive skewness. In Fig. 6‡ the curve has the special one-sided form of maximum positive skewness, while in Fig. 4† the mode is greater than the mean, producing so-called negative skewness. In these four cases the curvature indicates a lower limit greater than zero. When suitable values for these limits are subtracted from the observations and they are replotted they follow the dotted lines as shown. Hence, the required function for linear plotting for these examples is $(R - a)$.

In Fig. 5,§ the first plotting of the data is concave downward. This indicates that the theoretical frequency curve extends beyond zero to a negative limit. When the value of this limit is added to the original observations, the points can be plotted so as to approximate closely the auxiliary curve as drawn. As noted in the paper, the practical interpretation of this geometrical condition is that, in the case of this river, seasons of no run-off appear to be probable about 1.5% of the time, or about once in 65 to 70 years, and not that negative discharge is possible.

A final illustration of this type of curve with the lower limit greater than the zero, but with practically a maximum of skewness, will be given. Fisher‖ describes this as a "classical illustration" from the works of Professor Karl Pearson and uses it to illustrate the application of the very elaborate method of the "Gram-Charlier series" for the fitting of a curve to a very skewed frequency distribution by using "semi-invariants" and least squares, a method previously thought by some to be impossible in such cases. The problem may be solved graphically with the greatest ease and in a very short time, and the curve gives results directly which agree as well as those of Fisher and are much closer than those of Pearson. Furthermore, it is not even necessary to determine the equation of the curve to do this, although that can be written by simply finding the slope and intercept. The only constant that was required for the solution was the lower limit of the curve, and this

* *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1079.
† *Loc. cit.*, p. 1080.
‡ *Loc. cit.*, p. 1083.
§ *Loc. cit.*, p. 1081.
‖ "Mathematical Theory of Probabilities," by Arne Fisher.

was evident from the data themselves. Another reason for the use of this illustration was to show the method of preparing the data for plotting when they are given as a frequency distribution arranged from the statistics obtained from hundreds or thousands of observations.

TABLE 23.—Petal Flowers in Ranunculus Bulbosus.*

Number of petals in flower, x.	Number of flowers observed.	Percentage frequency.	Total percentage frequency, $\frac{100\,t}{n}$.	$x - 4$.	Total percentage frequency.	Total frequency.	Computed frequency.	Percentage of error.	Fisher.		Pearson.	
									Computed frequency.	Percentage of error.	Computed frequency.	Percentage of error.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
5	133	59.9	59.9	1	60.0	133.2	133.2	− 0.2	132.6	+0.4	136.9	− 3.9
6	55	24.8	84.7	2	84.8	188.2	55.0	0.0	55.2	−0.2	48.5	+ 6.5
7	23	10.4	95.1	3	94.3	209.2	21.0	+ 2.0	22.9	+0.1	22.6	+ 0.4
8	7	3.1	98.2	4	98.0	217.5	8.3	− 1.3	7.1	−0.1	9.6	− 2.6
9	2	0.9	99.1	5	99.3	220.3	2.8	− 0.8	3.0	−1.0	3.4	− 1.4
10	2	0.9	100.0	6	99.75	221.3	1.0	+ 1.0	0.1	+1.9	0.8	+ 1.2
..	222	....	.....	..	.....	.....	Average.	0.9	.....	0.6†	.....	2.7

* Data from the "Mathematical Theory of Probabilities," by Arne Fisher, p. 226.
† Using seven constants; for five constants the average error was 2.1 per cent.

The first and second columns of Table 23 give the number of petals and the number of flowers observed with the given numbers of petals. The total number of flowers examined (222) is the sum of the items in Column (2) and from this it is easy to compute the percentage frequencies given in Column (3). The progressive sums of these frequencies gives the total percentage frequencies in Column (4) which are used with the quantities in Column (1) for the trial plotting shown by the dots in Fig. 27. It is evident that a straight line will not fit these data, which follow a smooth curve concave upward. The data show that none of the flowers had as few as four petals although more than one-half the total number had five. The trial value for the lower limit, therefore, was made 4, and this was subtracted from the quantities in Column (1) and the differences set down in Column (5). These items were then plotted against the corresponding percentages in Column (4) as shown by the small circles in Fig. 27. The dotted auxiliary line was then drawn through the two lowest points, which should have the greatest weight, as the flowers with these numbers of petals had the greatest frequencies. It is evident that the line fits the data very well and a smooth curve can now be drawn through the original points as fully described in the paper, and indicated in Fig. 27, by plotting a series of points four units above the auxiliary curve.

It is interesting to compare the results obtained graphically with those obtained by the elaborate analytical methods. This can be done by multiplying the total percentage as read from the curve for each of the given

numbers of petals, by the total number, n, of flowers examined, which is 222. These quantities are set down in Columns (6) and (7). The first differences of the items in Column (7) are given in Column (8) and are the theoretical frequencies which may be compared with the observed frequencies given in Column (2) and also with the results obtained analytically by Pearson and Fisher which are given in Columns (10) and (12). The errors between the observed and computed quantities with their averages, taken without regard to sign, are given in Columns (9), (11), and (13). When the labor involved in the analytical methods used by either Fisher or Pearson, is compared with the ease and clearness of this graphical solution, the latter presents very definite advantages especially in the hands of those with practical judgment and experience in the use of graphical methods of computation and analysis.

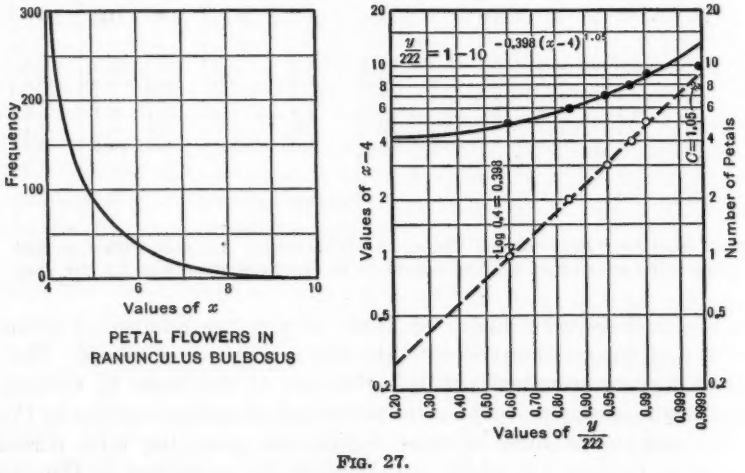

FIG. 27.

The curve (Fig. 27) shows that the probability of finding a flower with only four petals is zero, while there is only about one chance in a thousand of finding one with eleven or more petals. The graphical results have been compared with those obtained analytically by two different methods, both of which require the determination of three or more constants, while only two were used here, and both of these were obtained by inspection of the data. The first was the number of observations, n, while the second was the lower limit, a, of the frequency curve. (Fisher's very close results required the determination of seven constants. With five constants, his results are little better than those of Pearson, the average error being 2.1 per cent.) As previously stated, it is only necessary to take off the slope and intercept in order to obtain two additional constants and then to write down the equation of the curve. It is then possible to compute any of the theoretical values of any of the usual measures used for purposes of comparison with those obtained from the data or from other frequency series. The equations of this curve referred to the two ordinary systems are:

$$y = 222\,(1 - 10^{-0.398\,(x-4)^{1.05}})$$

and,

$$y = 222 (1 - e^{-0.916 (x - 4)^{1.05}})$$

The formulas for the constants and coefficients referred to as the usual measures of comparison, based on the equation with the lower limit, a, are given by the author's Equations (1)* and (5) to (9),† inclusive.

The next step in the generalization of the equation for these frequency curves, was to provide for the many cases when the data, as plotted on logarithmic or skew frequency paper, showed a reversal in the direction of curvature, pointing to or suggesting a finite upper limit. After trials with various combinations it was found that the purpose could be accomplished by putting the equation in the following forms:

$$t = n \left(1 - 10^{-h \left(\frac{R - a}{b - R}\right)^c}\right)$$

or,

$$t = n \left(1 - e^{-k \left(\frac{R - a}{b - R}\right)^c}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

In these equations, b is the finite upper limit while the other constants and variables are the same as before. One advantage in this system of frequency curves is that the constants have the same significance in all the equations whatever the limits and for all values of skewness. This is not the case with the Pearson curves for which the equations take widely differing forms for different limits and skewness.

Data showing this reversal of curvature are illustrated in Figs. 7‡ and 8. It is somewhat more difficult to fit curves to such data, since it is necessary to determine suitable values for both limits, a and b, before the linear function, $\left(\frac{R - a}{b - R}\right)$, can be plotted. While it is possible theoretically to obtain approximate values of these constants by graphical methods with data taken from preliminary frequency and duration curves, it is usually quicker and easier to find them by trial unless the data are so numerous and regular as to enable one to draw very smooth preliminary curves.

With a little practice it will be possible in most ordinary cases to select suitable values for the limits after a few trials and when these have been determined, small changes in the lower limit will alter the alignment at the lower end of the curve much more than at the upper, and *vice versa*.

In addition to the illustrations in the paper, the data in Table 24 afford a characteristic example of this same type. This table gives the annual maximum flood heights from 1912 to 1925, inclusive, on the Sze Ya Ho, at Ti Liu Pu, near the center of the Chihli Plain, in North China. In this table, G is the elevation of the maximum water level, in meters, above Taku Datum, as given in Column (1), with the corresponding percentages of the time in Column (2). The limits, 2 and 10, were used to compute the other quantities in Table 24 after three other combinations were also tried with limits of 1 and 9. The auxiliary curve (Fig. 28) can be located by eye so as

* *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1068.
† *Loc. cit.*, p. 1070.
‡ *Loc. cit.*, p. 1084.

to pass through one-half these points with the others scattered on both sides of the line, giving very satisfactory results. The curve shows that one-half the maximum flood heights are between Elevations 5.25 and 7.25, giving an average range of just 2 m. One-half the floods are higher than 6.4 m., and hence the skew ratio is — 0.30. As this is negative, the mode is greater than the mean, indicating that flood heights above the mean are more probable than those below.

TABLE 24.—Annual Maximum Flood Heights on the Sze Ya Ho, at Ti Liu Pu, North China, 1912-1925.

Maximum flood level, in meters, G. (1)	Percentage of the total time, t. (2)	$G - 2$. (3)	$10 - G$. (4)	$\frac{G-2}{10-G}$. (5)
3.2	3.6	1.2	6.8	0.18
4.6	10.7	2.6	5.4	0.48
4.8	17.8	2.8	5.2	0.54
4.9	25.0	2.9	5.1	0.57
5.6	32.1	3.6	4.4	0.82
5.6	39.3	3.6	4.4	0.82
6.3	46.4	4.3	3.7	1.16
6.6	54.5	4.6	3.4	1.35
6.6	60.7	4.6	3.4	1.35
7.0	67.9	5.0	3.0	1.67
7.6	75.0	5.6	2.4	2.33
7.6	82.2	5.6	2.4	2.33
7.8	89.3	5.8	2.2	2.46
8.0	96.4	6.0	2.0	3.00

Dangerous stages occur when floods rise higher than an elevation of about 7.5, which the curve shows to be probably about 16% of the time, or once in 6 or 7 years. Two such floods have occurred during the period of observation, once in 1917 when the City of Tientsin was greatly damaged and again in 1924 when the city was threatened for several weeks with a similar flood. Tientsin was flooded in a similar disastrous manner about thirty years prior to 1917, and it was generally thought that dangerous floods were probable only about once in that period, but investigations show that there might be as many as five or six dangerous floods on the plains west of the city during that period. The curve also shows that a flood as high as 8.5 m. above Taku Datum is not probable more than once in 100 years, while floods only ½ m. lower may be as frequent as once in 15 years.

If a straight line had been drawn through the points as originally plotted, the lower and middle portions of the curve would have been effected but little. If, however, this line were extended to estimate the probable occurrence of floods and their height from 1 to 10% of the time, very erroneous conclusions might have been reached, such as that a flood height of 9.0 m., or more, might be expected every 50 years. As the present river dikes have never been raised higher than Elevation 8.0, Taku Datum, they would all be overtopped or breached long before such an extreme elevation could be reached. They have been in existence for more than 100 years. Attention is called to these facts in order to emphasize the necessity of taking physical conditions into account even when dealing mathematically with such problems.

The "most probable" flood height is determined by the mode, for which the modulus and index must first be determined from the auxiliary curve (Fig. 28). The intercept is at the 40% ordinate and is, therefore:

$$-\log.\ (1 - 0.4) = 0.222 \text{ nearly.}$$

The slope of the auxiliary curve is 1.52 and hence the equation of the curve is:

$$\frac{t}{100} = 1 - 10^{-0.222\left(\frac{G-2}{10-G}\right)^{1.52}}$$

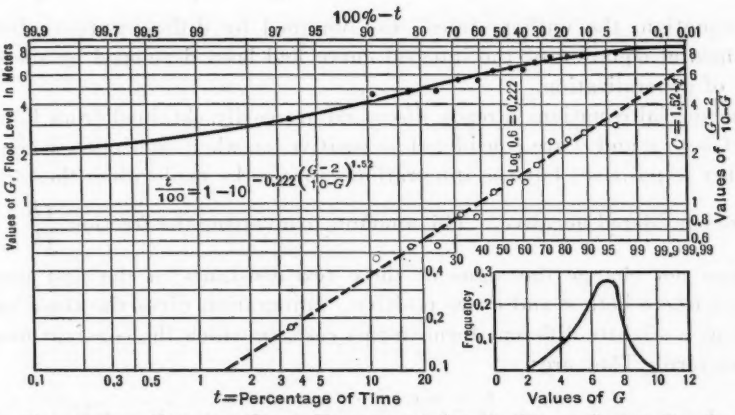

FIG. 28.—ANNUAL MAXIMUM FLOOD HEIGHTS ON SZE YA HO, AT TI LIU PU, CHINA.

A direct solution for the mode cannot be made for this equation, which involves two finite limits, but an approximate value can be determined by repeated trials, by substitution in the following equation:

$$(c - 1)\ (b - R) + (c + 1)\ (R - a) = \frac{c\ h}{M}\ (b - a)\ \frac{(R - a)^c}{(b - R)}$$

This equation can be simplified by putting the ratio, $\frac{(R - a)}{(b - a)}$, equal to z and solving for h. Thus;

$$h = M\left(\frac{1 - z}{z}\right)^c \left(1 + \frac{2z - 1}{c}\right)$$

A set of curves may be plotted from this equation. If the base, e, is used instead of the base, 10, it is only necessary to remember the relation, $k = h\,M$, and make the necessary substitutions. Entering such a diagram with corresponding values of h and c from the equation of the curve, an approximate value of z could be found. From this the mode may be determined by the formula:

$$\text{Mode} = z\ (b - a) + a$$

For the foregoing example, the mode is approximately 6.68 m. which is, therefore, the "most probable" flood height sought.

Although the special equations just given cover the majority of the more common frequency curves, data will sometimes be found which can not be adequately represented by these curves even when four constants are used,

in addition to n, which is usually determined at once by the data. However, there are still other very similar special equations derived from the same general frequency equation, which can be used in the same way on this skew frequency paper.

The general form of this empirical equation for skew frequency curves, using the base, e, is for the differential curve:

$$t' = n\,k\,[c\,(b - R) + d\,(R - a)]\left(\frac{R - a}{b - R}\right)^{\frac{c-1}{d+1}}\left[e^{-k\,\frac{(R-a)^c}{(b-R)^d}}\right] \ldots . (G')$$

This equation, the author states, was obtained by differentiation after the fundamental equation of the integral curve had been developed by successive stages of generalization.

The special equations already discussed are easily obtained from Equation (G') if both c and d are considered as positive, or when one index is zero and the other is positive; but the integration can also be made when these exponents are considered negative. This amounts to inverting the fraction, $\frac{(R - a)}{(b - R)}$, and does not change the signs of these two constants in the first term in brackets where both c and d are positive. Integration gives the three special equations a slightly different form in this case, in which the constant term, n, becomes zero. They are:

$$t = n\,e^{-k\,R^{-c}} \ldots . (T)$$

$$t = n\,e^{-k\,(R-a)^{-c}} \ldots . (D)$$

$$t = n\,e^{-k\,\frac{(R-a)^{-c}}{(b-R)}} \ldots . (E)$$

One or two illustrations will show when these equations should be used and how the data are handled. First, take an example from Yule (Table 25) which gives the mean annual death rate per 1 000 of population in certain districts in England.

TABLE 25.

Mean annual death rate per 1 000, R.	Number of districts with given death rate.	Frequency, percentage.	Total frequency, t, percentage.
12.5–13.5	5	0.79	0.79
13.5–14.5	16	2.53	3.32
14.5–15.5	61	9.65	12.97
15.5–16.5	112	17.73	30.70
16.5–17.5	159	25.13	55.83
17.5–18.5	104	16.45	72.28
18.5–19.5	67	10.60	82.88
19.5–20.5	42	6.65	89.53
20.5–21.5	25	3.96	93.49
21.5–22.5	18	2.85	96.34
22.5–23.5	8	1.27	97.61
23.5–24.5	5	0.79	98.40
24.5–25.5	3	0.48	98.88
25.5–26.5	1	0.16	99.04
26.5–27.5	1	0.16	99.20
27.5–28.5	2	0.32	99.52
28.5–29.5	....	.....	.....
29.5–30.5	....	.....	.....
30.5–31.5	2	0.32	99.84
31.5–32.5	....	.....	.....
32.5–33.5	1	0.16	100.00

When these data are plotted in the usual manner (Fig. 29), rising from left to right, the lower end is extremely flat while the points at the upper end fall very close together and rise too rapidly to be fitted by any of the curves previously used. The percentage ordinates are spaced much more widely at the left side of the diagram so that if the upper points were plotted at that side the curve would not bend upward so sharply. Accordingly, the data are replotted rising from right to left as shown in the upper curve of Fig. 28, using the percentage scale at the top of the diagram. A line drawn through the third and thirteenth points fits the intervening ones with remarkable exactness while departing but little from those at either end. The slope of this

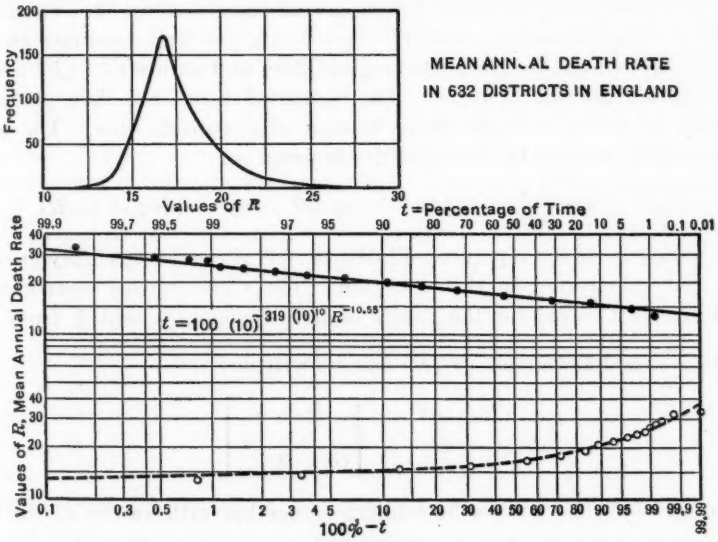
MEAN ANNUAL DEATH RATE
IN 632 DISTRICTS IN ENGLAND
Frequency
Values of R
t = Percentage of Time
Values of R, Mean Annual Death Rate
t = 100 (10)^319 (10)^10 R^-10.55
100% - t

FIG. 29.

line is evidently negative as compared with the previous diagrams, which indicates that Equation (T) will fit the case. As the intercept of this curve is outside the limits of the diagram, the values of the constants for the equation can best be determined by substituting the co-ordinates of two points on the curve in the equation written in the logarithmic form, and solving for the unknown modulus and index. The resulting equation is:

$$t = 100\ (10)^{-319\ (10)^{10}\ R^{-10.55}}$$

in which, t is the percentage of the total number of districts having the death rate, R. The number of districts, 632, may be substituted for 100 in the equation, and t will then give the approximate number of districts instead of the percentage. The most frequent rate will be given by the mode, but the formula for this, as well as for the standard deviation, etc., differs slightly from those given previously. For the mode and mean, the formulas for Equation (D) are:

$$\text{Mode} = a + \left(\frac{c\,k}{c+1}\right)^{\frac{1}{c}}$$

$$\text{Mean} = a + k^{\frac{1}{c}}\,\Gamma\left(1 + \frac{1}{c}\right)$$

Other equations can be written by analogy, but all the formulas which involve the gamma function fail for values of c that produce negative integral values of gamma because the gamma function then becomes infinite. Fractional negative values give finite values of the function.

Another illustration is that of the Wimera River at Glenorchy, Australia, shown in Fig. 11,* where the curve is plotted rising from right to left. Even the use of Equation (C)† with four constants does not give an entirely satisfactory curve, while plotting the data in the other direction is out of the question. By using the general Equation (F)‡ with five constants, in which, c and d are negative, a very good curve (considering the irregularity of the data) can be obtained, as shown by the dotted curve in Fig. 11. The solution is made in the following manner. The limits are first assumed at values which appear reasonable from the original data and values of $f(R)$ are computed with a slide-rule as usual. The computed ratios are then plotted and are found to follow a slight curve instead of a straight line. The general equation, when written in the logarithmic form, is:

$$\log\left(-\log\frac{t}{n}\right) = \log h + c\log(R-a) - d\log(b-R)$$

The co-ordinates of three points well distributed along the auxiliary curve, are chosen and substituted in this equation. The simultaneous equations thus formed are then solved for the values of the exponents, c and d, from which the ratio, $\frac{d}{c}$, is determined. $f(R)$ may be written:

$$\frac{(R-a)^c}{(b-R)^d} = \left[\frac{R-a}{(b-R)^{\frac{d}{c}}}\right]^c$$

Values of $(b-R)^{\frac{d}{c}}$ can now be computed, together with values of the linear function within the brackets in the right-hand member of the last equality. When these are plotted they will be found to follow a straight line much more closely and a new auxiliary curve can now be drawn. From this curve corrected values for the exponents and the modulus can be found as usual. In the equation for the final dotted curve of Fig. 11, it will be seen that the ratio, $\frac{d}{c}$, is $\frac{1}{3}$. Usually some such common ratio can be used to simplify the computations so as to use a slide-rule. In other cases a "log log" rule is very convenient.

The data for the annual run-off below the dam on the Sweetwater River, Calif., give an especially interesting curve. (See Table 11§ and Fig. 9.‖) Here, the lower limit of the curve is negative since years of no discharge are quite frequent. The plus (+) sign in the denominator of the linear function gives an upper limit of infinity instead of a finite one. The interpretation of

* *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1090.
† *Loc. cit.*, p. 1066.
‡ *Loc. cit.*, p. 1065.
§ *Loc. cit.*, p. 1086.
‖ *Loc. cit.*, p. 1087.

the frequency curve is particularly interesting and suggestive. The storage of water in the reservoir and the fact that it is used for irrigation through canals instead of being drawn off in the river bed, causes an accumulation of points near the lower limit. On the other hand, the reaction of this river to the causes producing years of large total run-off, seems to increase very rapidly with an increase in the intensity of those causes, as indicated by the long, nearly flat, right-hand portion of the frequency curve (Fig. 9) from 100 to 1 000% of the mean. Special attention is called to the three principal characteristics of this curve, as they are fully explained, in two instances at least, by particular physical conditions very evidently causing the finite frequency of zero run-off and the large number of years of small discharge due to the storage and use of water for irrigation. The causes producing the wide spread in the frequency curve for the years of large discharge are no less real although more obscure in the absence of complete information.

One of the most valuable features in frequency studies is the fact that, with reliable data, any departure from the ordinary shape or character of a frequency curve, points to some particular cause or causes which a careful study of conditions should disclose, thus making possible a more intelligent solution of the problem in hand.

For example, a report was made by Sigurd Eliassen, Assoc. M. Am. Soc. C. E., on the loess and silt carried by the rivers of North China. Frequency curves were shown for the percentage by volume of silt of varying diameters, as determined from a microscopical study of samples taken at three different stations of the Yung Ting Ho. All these curves were plotted with ordinary rectangular co-ordinates. The curve for the silt from the lowest station differed so widely from those for the two other stations that a question was raised as to the accuracy of all the curves. An analysis made on skew frequency paper, however, disclosed remarkable consistency as to the peculiar character of the distribution of the medium and larger diameters, while the comparatively low percentage of the smaller diameters in the curve for the lowest station was accounted for by the fact that those samples were taken from the river bed. At the upper stations all the samples had been taken from the stream during a high flood. Thus, they contained a certain quantity of very fine material which would not have a chance to settle and would be carried past the lower station. As a result, confidence in the curves was increased as they were shown to be fundamentally consistent.

Another and most unusual curve is shown in Fig. 17.* While the data themselves are of less interest to engineers, the curves are enlightening as the distribution is practically the opposite of that just described. For this curve, Equation (*C*) is used, which has minus infinity as the lower limit with a finite quantity as an upper limit. The frequency curve shows a considerable number of specimens of flax having zero length and a long flat curve for the frequencies of the shorter lengths of stalks with a large accumulation of items near the upper limit. Kapteyn's comments on this subject also show how the frequency curve can be interpreted in terms of the physical conditions surrounding the growth of the specimens under investigation.

* *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1100.

Another application of this graphical method of analysis is suggested by the possibility of resolving certain frequency distributions which show some peculiar irregularities into two (or more) simple and regular curves. The average annual rainfall at San Francisco and Sacramento, Calif., presents an interesting example of this kind. In the paper by C. E. Grunsky, Past-President, Am. Soc. C. E., on "Rainfall and Run-Off Studies",* the curve drawn through the points in his Fig. 4,† does not indicate any unusual characteristics. The records as combined in his Table 2,‡ cover a period of more than seventy years, and it is possible to draw a smooth curve through the points which averages them very well. When the data are plotted on skew frequency paper, however, as shown by the dots along the lower curve in Fig. 30, they follow a line that is approximately straight almost to the 90% ordinate where it has a pronounced upward trend, after which it again becomes nearly parallel to the lower portion. (On account of the large number and closeness of the points along the middle of the curve, only alternate ones are plotted from the 20% to the 85% time ordinates.) The points, 10% and 24% of the mean annual precipitation, were selected as the probable limits, and values of the linear function, $\frac{(R-10)}{(240-R)}$, were computed and plotted as shown by the circles along the upper curve. After numerous trials the two auxiliary curves (shown by the dashed lines, Fig. 30) were drawn to represent two independent frequency distributions. They have the same abso-

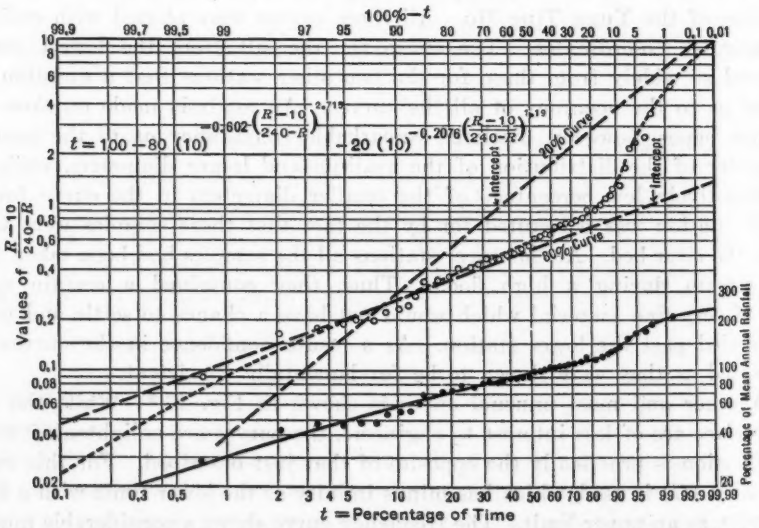

FIG. 30.—AVERAGE ANNUAL RAINFALL AT SAN FRANCISCO AND SACRAMENTO, CALIF.

lute limits, but quite different ranges within the practical limits of the diagram. The dotted auxiliary curve is formed from the two component curves

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 66.
† *Loc. cit.*, p. 79.
‡ *Loc. cit.*, p. 77.

by adding along horizontal lines for any given ratio, $\frac{(R-a)}{(b-R)}$, one-fifth of the reading for the line with the greater slope and four-fifths of that for the line with the lesser slope, to give the percentage of the total time for the corresponding point on the dotted curve. Thus, for a value of the ratio of 0.9:

$$\frac{1}{5} \times 34\% + \frac{4}{5} \times 93\% = 6.8\% + 74.4\%$$

or 81.2%, which gives the required point on the auxiliary curve for this value of the ratio. A series of points were found in this way and the dotted auxiliary curve was drawn, after which it was an easy matter to plot the curve through the original data in the lower part of Fig. 30.

From this analysis one is led to the tentative conclusion that, on the average, about 20% of the total annual rainfall is due to storms of a particular kind or class (which includes most of the heavier rains), while the ordinary precipitation, amounting to about 80% of the total, is of another and independent class or classes, probably differing materially as to origin or direction, or in some other way. The mean annual rainfall in Hawaii shows very similar characteristics.

A like conclusion has been strikingly confirmed within the past few months in connection with studies for the purpose of flood control on the Yung Ting Ho, one of the larger rivers in the Province of Chihli. This river has its head-waters and the greater part of its basin on the high plateau north and west of the Great Wall, beyond the mountain range which crosses the Province. The river passes this range through a long canyon and enters the plain near the capital, Peking, thence flowing across the plains before joining other rivers near the Port of Tientsin.

In making a graphical analysis of the gauge heights at the station nearest the mountains, the resulting frequency curve that most closely approximated the data, was a combination of two frequency curves very similar to those just described (Fig. 30). No single curve could be found even to approximate the requirements, although many trials were made previous to the resolution into two component curves. The conclusion thus indicated that two independent sets of causes producing run-off combine to bring about the distribution of river stages shown by the records, was confirmed by an independent investigation covering several years. This was made by Mr. Eliassen who has shown conclusively that rains in the northern part of China during the summer flood season are from two different sources. One class are cyclonic storms originating in areas of low barometric pressure, which form over Central and Western Mongolia and move in a southeasterly circle toward Japan. Other storms are of a local orographic character which develop along the coast and move in a northerly direction toward the mountains, while the most severe storms may also be classed as coastal, although they are caused by typhoons, which usually form in the South Pacific and move northward along the coast of China. These typhoons sometimes pass over parts of North China west of the Shantung Peninsula, although they usually pass to the

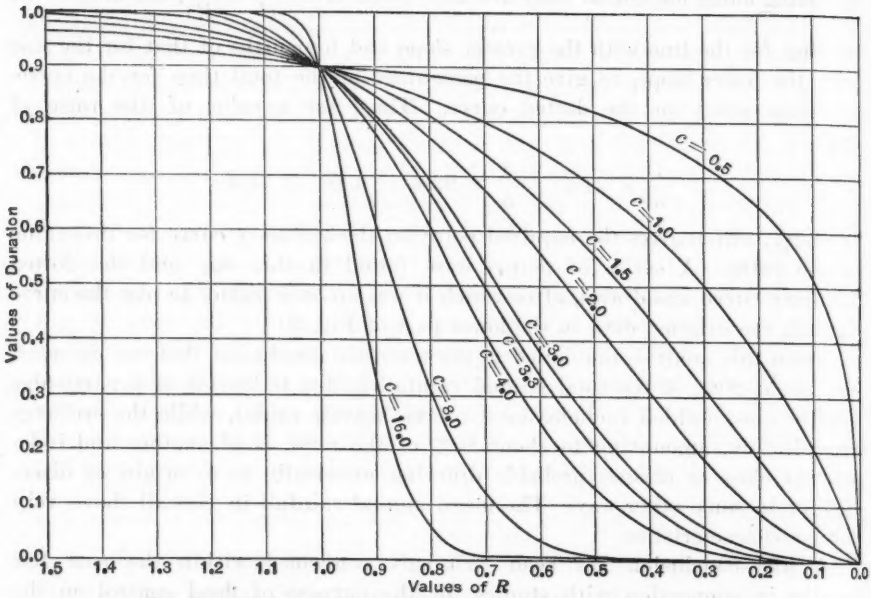

FIG. 31.—DURATION CURVES SHOWING EFFECT OF VARIATION IN c.

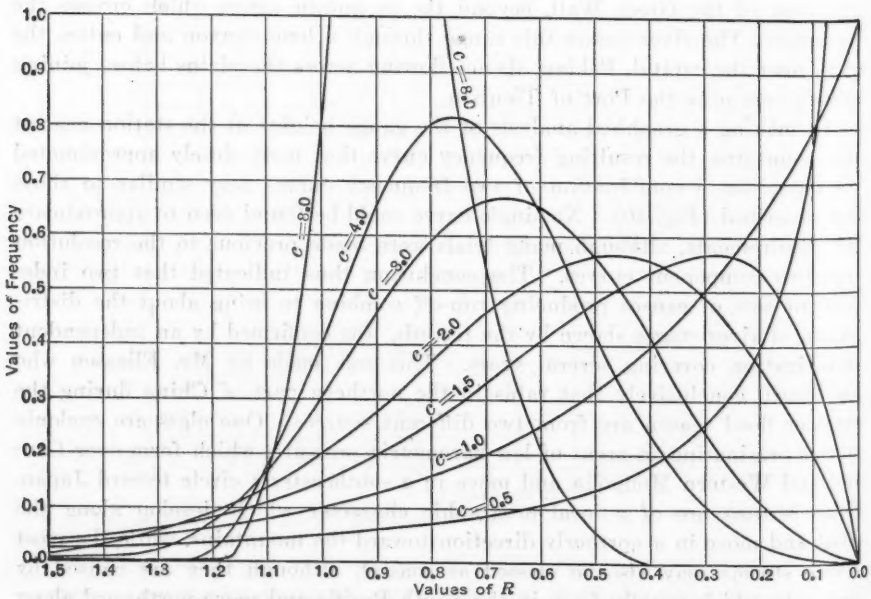

FIG. 32.—FREQUENCY CURVES SHOWING EFFECT OF VARIATION IN c.

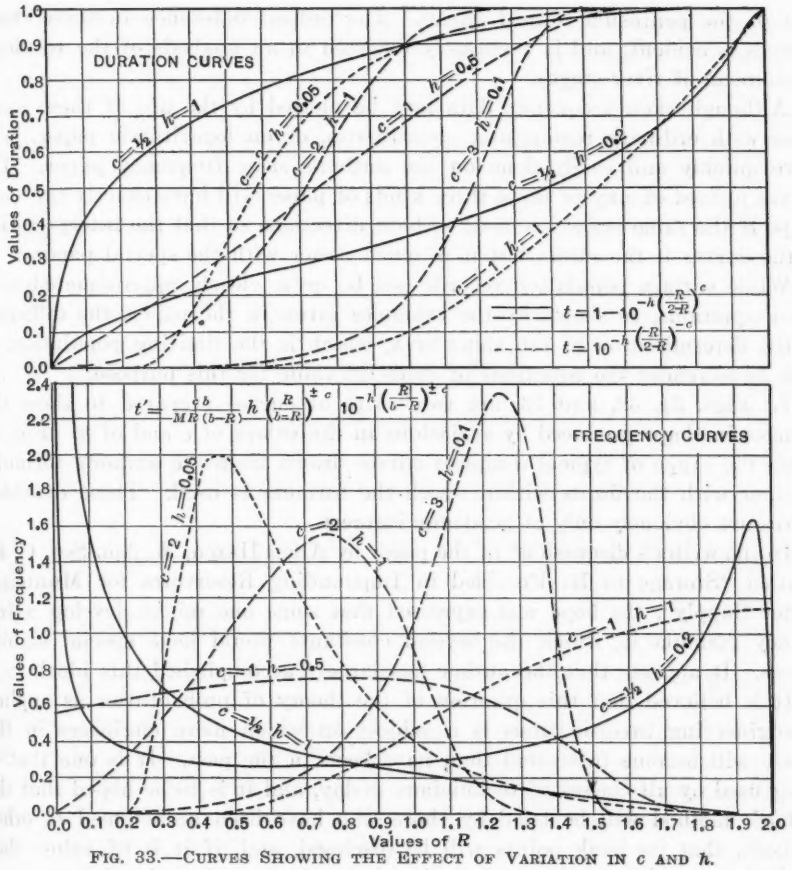

FIG. 33.—CURVES SHOWING THE EFFECT OF VARIATION IN c AND h.

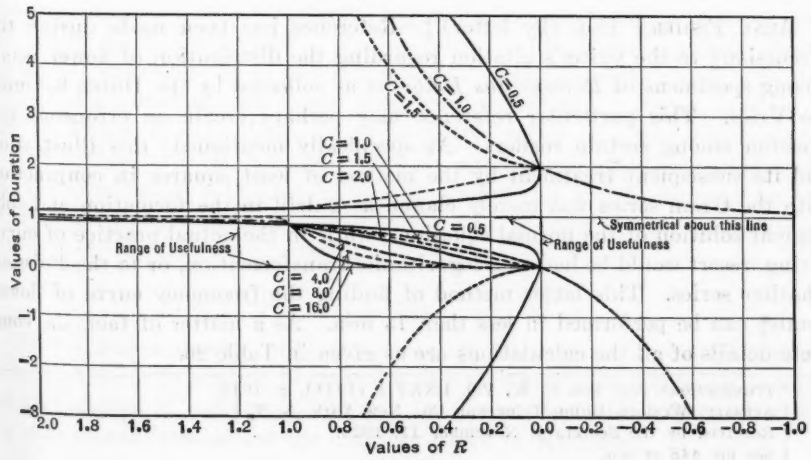

FIG. 34.

east of the peninsula toward Japan. The radical difference in these storm sources is evident, and is accurately reflected in an analysis of the resulting phenomena of river stages.

Although skew frequency data may be plotted by the use of these equations with ordinary rectangular co-ordinates, or on logarithmic paper, it is more quickly and easily done on the author's skew frequency paper. The curves plotted on any of these three kinds of paper will have exactly the same shape if the same scales are used in both directions so that the interpretation of the curves is the same; but it is much easier with the special paper.

While certain population records can be quite closely approximated with these equations, as shown by the examples given in the paper, the difficulty in the determination of the value of n, which is the limiting population, is such as to render the equations of doubtful value for this purpose.

In Figs. 31, 32, and 33, are two series of curves prepared to show the changes in shape produced by variations in the values of c and of h. Fig. 34 shows the shape of typical complete curves drawn from the author's formula, together with the limits within which the formula is used. These extended curves are obviously only of academic interest.

In the writer's discussion* of the paper by Allen Hazen, M. Am. Soc. C. E., entitled "Storage to Be Provided in Impounding Reservoirs for Municipal Water Supply", the hope was expressed that some one might develop a frequency formula in which the several constants would have special significances. It appears that the author has largely accomplished this ideal.

It is believed that this question of the theory of probabilities as applied to engineering investigations is a subject on which more engineers in the future will become interested than have been in the past. It is one that is being used by all classes of technicians to-day, and it is to be hoped that the author's method will be tried by those who have been accustomed to other methods, that its weak points will be disclosed, and, if it is of value, that it will be as widely used as it is believed that its actual merits deserve.

ARNE FISHER,† ESQ. (by letter).‡—Reference has been made during the discussion§ to the writer's citation regarding the distribution of flower petals among specimens of *Ranunculus Bulbosus* as collected by the Dutch botanist, De Vries. This particular reference may perhaps create an erroneous impression among certain readers. As specifically mentioned‖ this illustration and its subsequent treatment by the method of least squares in conjunction with the Gram series was merely chosen as a drill in the formation and subsequent solution of the normal equations, while in the actual practice of curve fitting, resort would be had to a logarithmic transformation, or to the Poisson-Charlier series. This latter method of finding the frequency curve of flower petals¶ can be performed in less than 15 min. As a matter of fact, the complete details of all the calculations are as given in Table 26.

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* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1646.

† Actuary, Western Union Telegraph Co., New York, N. Y.

‡ Received by the Secretary, November 11, 1926.

§ See pp. 448 *et seq.*

‖ "The Mathematical Theory of Probability," by Arne Fisher, p. 226.

¶ *Loc. cit.*, p. 274.

TABLE 26.—SCHEDULE FOR COMPUTING THE POISSON-CHARLIER FREQUENCY CURVE FOR THE DISTRIBUTION OF PETAL FLOWERS IN RANUNCULUS BULBOSUS.

No. of petals.	x.	Observed frequency, $O(x)$.	$x\,O(x)$.	$x^2\,O(x)$.	Poisson-Charlier, frequency curve, $F(x)$.
5	0	133	0	0	134.5
6	1	55	55	55	51.6
7	2	23	46	92	22.5
8	3	7	21	63	9.5
9	4	2	8	32	2.9
10	5	2	10	50	0.6
..	..	222	140	292	221.6

Values for the different quantities will then be as follows:

$$\lambda_0 = c_0 = 222;\ \lambda_1 = m = 140 : 222 = 0.631;$$

$$\lambda_2 = 292 : 222 - \lambda_1{}^2 = 1.315 - 0.398 = 0.917;$$

$$c_2 = \frac{1}{2}(\lambda_2 - m) = \frac{1}{2}(0.917 - 0.631) = 0.143;$$

and,

$$F(x) = c_0\,[\phi_m(x) + c_2\,\Delta^2\,\phi_m(x)] = 222\,[\phi_m(x) + 0.143\,\Delta^2\,\phi_m(x)]$$

in which, $\phi_m(x) = \dfrac{e^{-0.631}\,0.631^x}{x!}$ is the usual Poisson probability function of the form, $e^{-m}\,m^x : x!$, for which extensive tables with first and second differences have been published for various values of m from 0.1 to 15.5. It takes but little experience with the theory of frequency curves to convince any competent investigator that this method is superior to the quasi-analytical graphical process suggested by the author.

There are two distinct uses to which the theory of frequency functions has been put, one being merely that of a purely empirical graduation process, and the other, that of arriving at the probable or presumptive values of the statistical parameters of future observations. The paper deals only with the subject of graduation and leaves almost untouched the far more subtle and difficult aspects of the probable or presumptive values of future observations.

The author states most frankly that his equations are "entirely empirical". His problem seems to be that of finding a relatively simple formula for the integral of a generalized frequency curve, and he suggests as a solution to this problem the following formula:

$$t = m - n\,(10)^{-h\frac{(x-a)^c}{(b-x)^d}}$$

This equation appears to be a special case, although in a greatly modified form, of Quiquet's general formula for the "integral" (or "duration") curve for the number of survivors, $l(x) = t$, in a mortality table. Quiquet writes his generalized formula for the duration curve in the form:

$$\log l(x) = A + Bx + \sum e^{k_i x} f_i(x)$$

in which, the k's, are certain constants and $f_i(x)$ polynomials possessing some important properties which have been discussed in detail.*

As pointed out by the author the Dutch mathematicians, Kapteyn and Van Uven, have also suggested similar forms of equations in their two well-known papers on "Skew Frequency Curves in Biology and Statistics". For the benefit of readers interested in the subject it may be stated that two later publications by Van Uven, dealing with skew frequency curves and logarithmic frequency distributions, were published during 1916.*

While Van Uven's method is based on a sound and rigorous theoretical foundation (namely, that of finding the function, $h(x)$, of the variable, x, which will be normally distributed, although the variable, x, itself is not normally distributed), the author's purely empirically derived formulas appear to be nothing more than very neat graduation formulas.. As such they are most valuable if one is satisfied with graduating certain observations by the particular graphical process described in the paper; but as probability formulas for presumptive frequency curves and as a basis for the forecast of future events or observations, they seem open to various objections.

The several constants in the author's formula are, like all empirically derived values, subject to sampling errors and it seems that one of the paramount requirements would be to furnish reliable formulas for determining the standard (or probable) errors of these constants. For instance, if it should be found that a constant has the numerical value of, say, 0.0345, with a standard error of ± 0.105, there are evidently no good reasons for attaching any importance to the constant itself.

The author does not furnish any formulas for the determination of such standard errors, and the writer has not been able thus far to work out such formulas himself. On the other hand, all the numerical examples in the paper may be analyzed and graduated by means of expansion in Gram's series where the generating functions may be either the Laplacean normal probability function, the Poisson probability exponential, the Kapteyn-Van Uven functions, the frequency functions of Pearson, or any other generator fulfilling certain simple *a priori* requirements. Since all the constants in the Gram series can be expressed as semi-invariants for which one can compute the standard errors according to the formulas originally derived by Thiele,† it may be of interest to contrast this latter method briefly with the one proposed by the author.

As an illustration take the observations in the author's Table 4‡ on the annual rainfall in Tientsin, China. These observations, thirty-two in number, may be arranged in a frequency table of the form shown in Table 27.

A calculation of the first four semi-invariants gives the following results (interval = 3 in., provisional mean at 19):

$$\lambda_1 = 0.031$$
$$\lambda_2 = 3.147$$
$$\lambda_3 = 1.425$$
$$\lambda_4 = -6.725$$

* *Bulletin trimestriel* des Actuaries Français, 1893.

† Koninklijke Akademie van Wetenschappen te Amsterdam, Vol. 19 (1916).

‡ Almindelig Iagttagelseslaere," Copenhagen, 1888.

§ *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1078.

from which,

$$\text{Mean} = M = 18.907 \text{ in.}$$
$$\text{Dispersion} = \sigma = 5.322 \text{ in.}$$
$$\text{Skewness} = c_3 = -0.0425^*$$
$$\text{Excess} = c_4 = -0.283$$

TABLE 27.

Rain, in inches.	x.	Frequency.
9–12	− 3	2
12–15	− 2	5
15–18	− 1	7
18–21	0	5
21–24	+ 1	6
24–27	+ 2	4
27–30	+ 3	2
30–33	+ 4	1
Total.....	....	32

The Gram series is, therefore, of the following form,

$$F(x)\,dx = [\phi_0(x) - 0.0425\,\phi_3(x) - 0.0283\,\phi_4(x)]\,dx$$

in which,

$$\phi_0(x) = \frac{1}{5.322\sqrt{2\pi}}\,e^{-\frac{1}{2}\left[\frac{x-18.907}{5.322}\right]^2}$$

and $\phi_3(x)$ and $\phi_4(x)$ are its third and fourth derivatives for which the numerical values are given in the well-known standard table by Jörgensen or in the smaller table by Glover.

The author's formula for the "duration" curve of the rainfall is:

$$\frac{t}{100} = 1 - (10)^{-0.00193\,(x-8)^{2.10}}$$

A comparison between the observed frequencies and the respective graduations according to the two methods is given in Table 28.

TABLE 28.

Rainfall, in inches $= x$. (1)	Observed frequency. (2)	Goodrich's formula. (3)	Gram's series. (4)	Normal curve of the logarithms of observed rainfalls. (5)
Less than 9	0	0.14	0.64	0.1
9–12	2	2.37	2.43	1.8
12–15	5	4.92	4.81	5.0
15–18	7	6.28	6.43	7.0
18–21	5	6.16	6.43	6.6
21–24	6	5.00	5.26	5.0
24–27	4	3.42	3.50	3.0
27–30	2	2.00	1.76	1.7
More than 30	1	1.71	0.72	1.8

Testing for "goodness of fit" of these graduations in accordance with the Pearson criterion (which criterion—or its essentials—by the way had already

* In Pearson's notation the skewness equals $-3c_3$, and excess (or kurtosis) equals $3c_4$.

been found by Helmert in the Seventies), it is found that $P = 0.9965$ in the case of the Goodrich formula, and $P = 0.9914$ for the Gram series, indicating an almost perfect and practically equally good fit for the two methods of graduation. It must be constantly borne in mind, however, that these results are only graduations pure and simple and do not give any inkling about the frequencies of future rainfalls.

As stated, the writer does not know of any method of measuring the standard errors in the Goodrich formula; but in the case of the Gram series the methods as developed by Thiele show that the standard errors for the skewness, c_3, and the excess, c_4, exceed ± 0.075 and ± 0.036, respectively.

All this goes to show that one is not justified in attaching any importance to the measures of skewness and excess (or, for that matter, to the closeness of "goodness of fit"), since the standard errors of these constants are greater than the constants themselves in the small sample of thirty-two items. Moreover, since Gram's series yields just as good a fit as the Goodrich formula, there seems good reason to demand that the author furnish standard errors for his empirically derived constants before attaching any undue importance to his results. In fact, the writer seriously questions whether the proposed method in meager samples of this kind gives more reliable results than the elementary method of finding the mean and the dispersion of the logarithms of the individual observations in conjunction with the use of the ordinary probability integral as shown in Column (5), of Table 28, which represents the formula, $\frac{32\,dz}{\sqrt{2\pi}}\,e^{-z^2:2}$ when $z = (\log x - 1.275) : 0.1273$.

As a second illustration take the frequency records of two series of twenty-six consecutive winnings each in a carefully performed game of chance, as given in Table 29.

TABLE 29.

Winnings, 10 R.	Frequency, in first series.	Frequency, in second series.
\$0.40	1	0
0.50	1	0
0.60	1	0
0.70	4	1
0.80	4	8
0.90	4	5
1.00	3	2
1.10	3	2
1.20	1	4
1.30	2	1
1.40	2	2
1.50	0	1
Total.....	26	26

Through an application of the graphical method, as suggested by the author and illustrated in Fig. 35, the following results were obtained for the "duration" curve for the first series:

$$\frac{t}{100} = 1 - (10)^{-0.00002235\,(10\,R - 40)^{2.354}}$$

The frequency curve proper corresponding to the duration curve is shown in graphical form in Fig. 36. No attempt has been made to fit the second series to the Goodrich formula since it is evident that quite a different "duration" curve from the one in the first series would be obtained.*

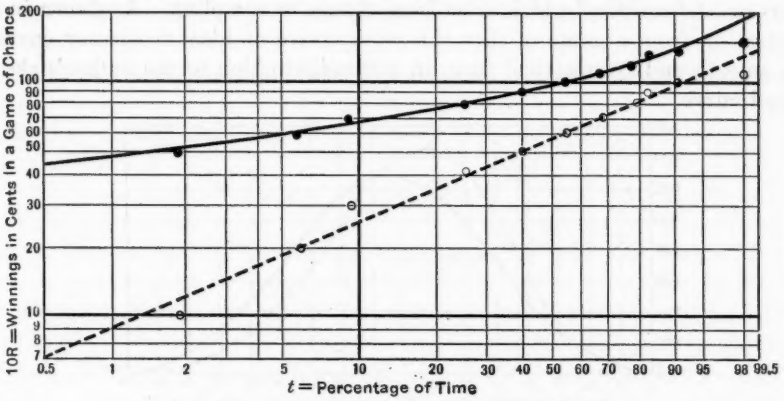

FIG. 35.—GRAPHICAL FORM OF DURATION CURVE.

What are the real facts in this illustration? The series previously discussed represents the monetary winnings in the following simple game of chance, namely, three dice are thrown simultaneously and the winnings equal ten times the sum of the results of the throw. In other words, one is faced with the classical problem of Galileo in a slightly modified form. There are altogether 216 possible cases of sums in a single throw of three dice, running from 3 to 18, the former being the event of obtaining 3 ones and the latter being the event of obtaining 3 sixes. The true *a priori* probabilities for the different winnings are as given in Table 30.

The probability curve (Fig. 36) can be expressed very closely by the following Gram series:

$$F(x)\,dx = [\phi_0(x) - 0.017\,\phi_4(x)]\,dx$$

in which,

$$\phi_0(x) = \frac{1}{2.958\sqrt{2\pi}}\,e^{-\frac{1}{2}\left[\frac{x-11.5}{2.958}\right]^2}$$

is the generating function and $\phi_4(x)$ its fourth derivative.†

This true presumptive curve is shown alongside the entirely differently shaped Goodrich curve in Fig. 36 and demonstrates beyond doubt that the author's statement‡ to the effect that, "the graphical analysis of 'integral skew frequency' or 'duration curves', as herein presented, is developed espe-

* It should be emphasized that the results of the two series as shown in Table 29 are due solely to chance and are not caused by any physical defects in the dice themselves. In fact, the two series represent the two first samples of 40 total samples, each sample containing 26 individual throws. The 40 samples when treated as a generalized Bernoullian series showed perfect agreement with the *a priori* theory. The empirical results produced a skewness which was about equal to its standard error, while the excess (kurtosis) came within 5% of that of the *a priori* theory. All this goes to show that the dice were not "loaded," and that they were thrown in a purely random fashion. The true skewness is, of course, zero, since the curve is symmetrical; the empirically derived skewness of the total 1040 throws was —0.0012.

† Since the presumptive curve is symmetrical around the mean, the skewness is equal to zero and the excess (kurtosis) as expressed by the semi-invariants of even order need only be considered.

‡ *Proceedings*, Am. Soc. C. E., August, 1926 Papers and Discussions, p. 1104.

cially for the examination of records consisting of from about ten to fifty observations", is a bit premature, to say the least. All that a graphical process like that of the author does in the case of small samples is to yield a curve which fits the observations but makes no adequate allowance for the errors or "deformities" which arise from chance or sampling. As shown in the illustration from a game of dice the presumptive or ideal frequency curve is of a smooth and symmetrical form in contradistinction to the author's skewly shaped curve.

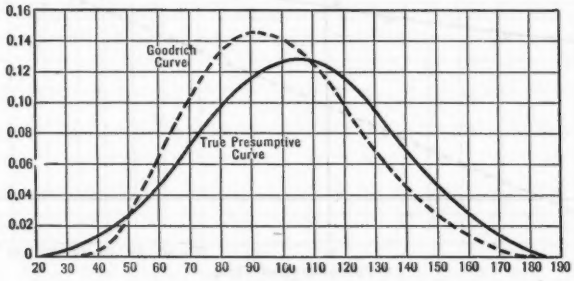

FIG. 36.—GRAPHICAL FORM OF FREQUENCY CURVE.

TABLE 30.

Winnings, 10 *R*.	Probability.
$0.30	1 : 216
0.40	3 : 216
0.50	6 : 216
0.60	10 : 216
0.70	15 : 216
0.80	21 : 216
0.90	25 : 216
1.00	27 : 216
1.10	27 : 216
1.20	25 : 216
1.30	21 : 216
1.40	15 : 216
1.50	10 : 216
1.60	6 : 216
1.70	3 : 216
1.80	1 : 216

As the writer passes on he merely suggests that the reader consider what would happen to a gambling establishment if it were to arrange the stakes in a game of chance, such as described in the previous paragraph, on the basis of the results as derived from an application of Mr. Goodrich's method.

In a sense Mr. Goodrich is in a position analogous to that of a dressmaker trying to fit a custom or tailor-made dress to a handsome woman. The woman, however, may have some small deformities; she may be slightly bowlegged, or have a slight stoop, or a hundred of other smaller imperfections. In a sense all these imperfections are errors or deviations from the ideal form, and woe to the tailor who would have the audacity to try to fit the lady's dress so snugly as to bring out and actually accentuate such errors. Yet, that is exactly what the author has done, and the use of his graphically derived curves as exact, or even as close, representations of the true or presumptive frequency functions has no sound mathematical basis unless one is able to evaluate the standard errors in his formulas.

In the case of the Gram series it is, on the other hand, a relatively simple process to evaluate the standard errors of the semi-invariants according to the methods as devised by Thiele. These methods when applied to the series of dice throws mentioned, indicate that the standard errors, not only of the skewness and the excess, but of the mean and the dispersion as well, are subject to sampling errors to such an extent that one should be extremely cautious in using the observed results for future predictions. In this connection it might perhaps not be amiss to mention the recent researches on variations in small samples, as carried on by the English biometricians, notably Dr. R. A. Fisher and "Student".

These researches, interesting as they are, do not in the writer's opinion approach and measure up to the more general methods by Thiele. Moreover, most of the methods of the English school are limited in the scope of their practical applications on account of the fact that they implicitly assume that the infinite universe from which the sample is drawn is normally distributed in respect to the variable attribute, an assumption which very rarely is realized, except possibly in the case of repeated precision measurements. Recently, attempts have been made to use rather refined mathematical methods in the case of as few as four or five observations. This, however, is a waste of time and is typical of the erroneous ideas among the adherents of the purely empirical school of statistics. As pointed out by Gram, when one deals with only four or five observations, it is not only easier, but also more exact, to deal directly with each individual result than to try to compress the observed facts into a mathematical formula involving the mean and the semi-invariants of higher order.

Many of the assertions by Dr. Fisher, in his exceedingly interesting and stimulating "Statistical Methods for Research Workers in Biology", do not rest on a sufficiently rigorous basis. At least in applying his own proposed method for the construction of mortality tables the writer has found that the criterions by Dr. Fisher under-estimate chance errors in small samples, and that it becomes necessary to use the more laborious, but also more powerful, criterions developed by Thiele and which he has characterized by the suggestive name of "Fejlkritik". This "Fejlkritik" is based on the hitherto little known and less appreciated theory of Thiele in connection with his formulas for the standard errors of the semi-invariants of the observations.

ALLEN HAZEN,* M. AM. SOC. C. E. (by letter).†—This paper presents a new method that is entirely graphical, for solving problems which, under the procedures previously suggested, were partly numerical and partly graphical. The procedures referred to were that of the writer‡ and that of H. Alden Foster, Assoc. M. Am. Soc. C. E.§ It may be taken for granted that any of these methods applied to reliable data will produce reliable results for points within the range covered by these data. Within this range the calculations may be classed as statistics, as distinguished from estimates for

* Cons. Engr. (Hazen & Whipple), New York, N. Y.
† Received by the Secretary, January 12, 1927.
‡ *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 214.
§ *Loc. cit.*, Vol. LXXXVII (1924), p. 142.

values outside the data ranges where extrapolation is an essential part of the process.

It is also to be mentioned that the method used by Weston E. Fuller, M. Am. Soc. C. E.,* was entirely graphical, but less general in character, and related only to the analysis of values near the upper end of the curve.

It will be shown how these four methods compare in computing, for example, the required spillway capacity to meet remote contingencies, when a record of moderate length only is available as a basis of calculation. This is a case of extrapolation and is the severest test of the methods.

The basis of the calculation is to be a record of the greatest 24-hour flows of the given stream through a period of years. These greatest 24-hour flows form series with decided skews. The flood ratio for each year is the ratio of the flood for that year to the mean flood for the entire record period. It would be possible to take an actual series for this comparison; such, for instance, as the record of the Merrimac River floods,† but for this purpose take assumed values for a 25-year series, these values being as nearly as possible normal or average values for data of this kind for the Northeastern United States. The values selected in this way are as follows:

2.02, 1.67, 1.50, 1.39, 1.30, 1.23, 1.18, 1.13, 1.09, 1.04, 1.00, 0.97, 0.93, 0.90, 0.87, 0.84, 0.81, 0.77, 0.74, 0.71, 0.67, 0.64, 0.60, 0.54, 0.46.

The coefficient of variation of this series is 0.37, and the coefficient of skew computed by the method described‡ by the writer is 0.91. As the coefficient of skew depends on the number of terms in the series, the skew to be used in applying Foster's method is to be adjusted as described by the writer,§ and the adjusted value to be used with the Foster factors is 1.22.

With this as a starting point, the writer has calculated from this assumed series the most probable values for points near the limits, by four methods as given in Table 31.

TABLE 31.

Percentage of time.	10	1	0.1	0.01
Computed by the writer, using the Goodrich graphical method	1.50	2.20	2.75	3.30
Foster method	1.50	2.17	2.78	3.35
Semi-logarithmic paper, Fuller method	1.50	2.25	3.01	3.77
Logarithmic probability method preferred by the writer, done either graphically or numerically	1.50	2.23	2.98	3.85

In Table 31 the results are arranged in the order of size, the Goodrich method giving the lowest values and practically checking the Foster method. The semi-logarithmic paper and the logarithmic probability paper form another pair with results not far apart and higher at the extremity.

Unfortunately, there are no data that can be used in determining which of these four methods is nearest the truth. That must be reserved for further study.

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 564.

† *Loc. cit.*, Vol. LXXXIV (1921), p. 220.

‡ *Loc. cit.*, p. 215.

§ *Loc. cit.*, Vol. LXXXVII (1924), p. 175.

ANALYSIS OF CONTINUOUS CONCRETE ARCH SYSTEMS

Discussion*

By Hardy Cross, M. Am. Soc. C. E.

Hardy Cross,† M. Am. Soc. C. E. (by letter).‡—This paper has real design value. The importance of studies in continuous arches is beginning to be realized, but there is still a wide gap between methods of analysis for general studies of such structures and design methods that will lead to better proportioning. In estimating the effect of continuity the changes in the horizontal reaction are the most important consideration, the rotation of the pier tops being usually of minor value. If the piers are relatively stiff, studies of three spans only are sufficient.

General Form of Influence Lines for Continuous Arches.—The outstanding questions are those that usually occur in indeterminate structures, in the following order:

1.—When and where should such structures be built?
2.—What are the proper proportions of piers, span, and rise?
3.—What points are critical in the design of such structures?
4.—How are these problems most readily analyzed?

As usual, the sequence of study is necessarily in reverse order to the questions.

Influence Lines.—Qualitative Studies.—As for the general action of arches on slender piers, the influence line for the crown moment, say, may be thought of as made up of three parts:

(*a*) The influence line for the fixed arch;
(*b*) The influence line for the effect of deflection of the pier tops; and,
(*c*) The influence line for the effect of rotation of the pier tops.

The effect of pier rotation is less pronounced than that of pier deflection. A combination of Parts (*a*) and (*b*) gives an influence line for the crown moment as shown in Fig. 23 (*a*). It is evident that the piers are pulled inward in all spans on either side of the span considered, causing the arch ring to rise (plotted down for graphic reasons) in the span considered and to drop in all spans on either side (negative influence ordinate). The curve of rise or fall, if it is assumed there is no rotation of the pier tops, has the shape of an

* This discussion (of the paper by C. S. Whitney, M. Am. Soc. C. E., published in May, 1926, *Proceedings*, but not presented at any meeting of the Society) is printed in *Proceedings* in order that the views expressed may be brought before all members.

† Prof. of Structural Eng., Univ. of Illinois, Urbana, Ill.

‡ Received by the Secretary, January 6, 1927.

influence line for crown thrust, but the ordinates become smaller outward from the pier considered.

Similarly, Fig. 23 (*b*) shows the two important constituent parts of the influence line for moment at the springing line, the influence line for fixed-ended conditions, and that for the effect of the reduction in crown thrust. This, again, shows the tendency to produce an influence line with all ordinates positive outside the span considered.

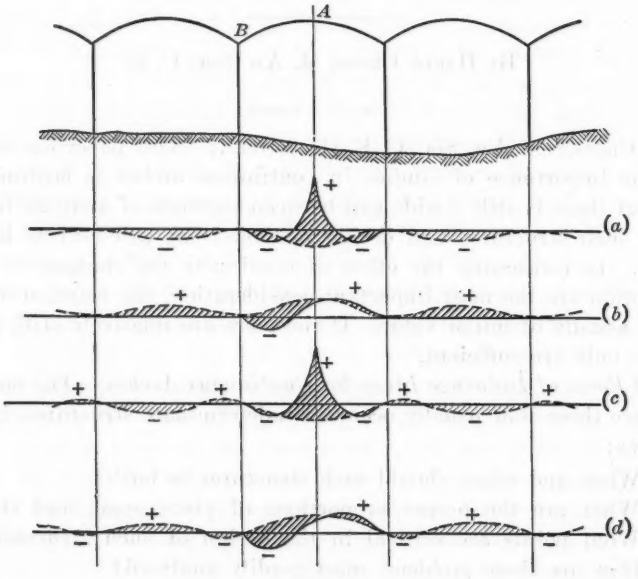

FIG. 23.

It is not difficult to see that the effect of pier rotation will be to modify Fig. 23 (*a*) and Fig. 23 (*b*) somewhat as shown in Fig. 23 (*c*) and Fig. 23 (*d*).

The Approximate Method of the Author.—The approximate method given by Mr. Whitney for determining the change in crown thrusts seems to the writer the most valuable part of the paper. This method consists of assuming the change in span in the arches to be due entirely to a horizontal thrust along the horizontal neutral line of the arch and that in the pier, to a thrust along this same line; and then of equating the displacements at the pier top. This assumes that the springing line of the arch does not rotate, but that the top of the pier is free to rotate. This is inconsistent, but seems to give good results.

As the author points out, a tremendous amount of work is necessary to determine the limits within which this approximation is satisfactory and also the limits within which the consideration of only three spans is satisfactory. He suggests that the criterion as to the value of the first assumption is to be found in the value, $(h + b - s)$. The writer has no good substitute to offer, and yet he ventures the opinion that the definition of the terms used in this expression might be made simpler. The values of h (height of pier) and of b

(distance from springing line of arch to horizontal neutral line) are clear enough, but s is defined by a long expression for $\frac{\phi D H}{\phi D M}$.*

The writer has roughly applied this approximation to continuous arch systems on which he has collected data, and had hoped to compare systematically the exact with the approximate results in these cases. A comparison of influence ordinates for crown moment in one case, however, is given in Fig. 24.

Apparently, in practical cases, the approximation indicated may well be used for preliminary design, the final design being checked by more exact methods. While the effect of live load is more pronounced in continuous arches than in fixed arches, the live load stress is less than one-half the total stress and the effect of errors is diminished accordingly.

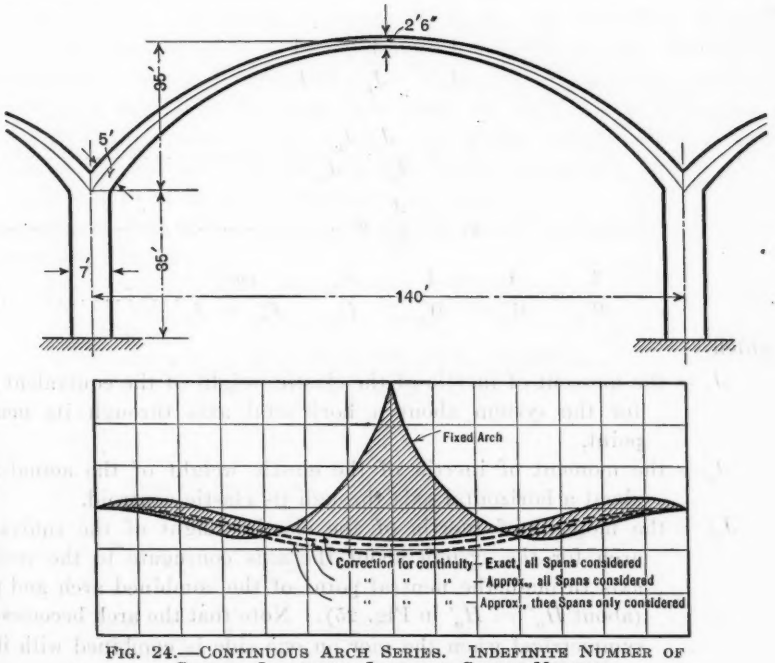

FIG. 24.—CONTINUOUS ARCH SERIES. INDEFINITE NUMBER OF SPANS. INFLUENCE LINE FOR CROWN MOMENT.

Exact Method of Analysis.—The exact analysis indicated by the writer† and referred to by the author,‡ is based on the idea that the action of any elastic body under the influence of terminal forces can be completely described in terms of (*a*) the co-ordinates of the center of rotation of this terminus for moments applied to it; (*b*) the parameter of the value of this rotation for a unit terminal moment; (*c*) the horizontal displacements of this center for a unit horizontal force and for a unit vertical force acting through this center;

* *Proceedings*, Am. Soc. C. E., May, 1926. Papers and Discussions, p. 845.
† *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1196.
‡ *Proceedings*, Am. Soc. C. E., May, 1926, Papers and Discussions, p. 878.

and (d) the corresponding vertical displacements of the center for unit vertical and horizontal forces through the center.

Determine the elastic properties of the end span, combine this with the second arch, and proceed thus until the arch under consideration is reached. Proceed similarly from the other abutment to the span in question. This gives the elastic properties of the arch under consideration and of two equivalent elastic piers on which it is regarded as resting. The solution for stresses in the span then may be made by the usual methods.

This gives an exact and definite procedure; the expressions used are not very formidable and may be simplified without appreciable loss of accuracy to give convenient solutions. The properties of each arch in the series are systematically determined and from these the stresses in the various arches.

The fundamental equations for the reduction process in the exact analysis are:

$$\frac{1}{J_s} = \frac{1}{J_p} + \frac{1}{J_a'}$$

or,

$$J_s = \frac{J_a' J_p}{J_a' + J_p} \quad \ldots\ldots (44)$$

$$y_p = \frac{J_s}{J_a'} y' \quad \ldots\ldots (45)$$

$$\frac{1}{W_s} = \frac{1}{W_a} + \frac{1}{W_p} + \frac{x^2}{I_a} + \frac{(y')^2}{J_a' + J_p} \quad \ldots\ldots (46)$$

in which

J_s = the moment of inertia of the elastic weight of the equivalent pier for the system about a horizontal axis through its neutral point.

J_p = the moment of inertia of the elastic weight of the actual pier about a horizontal axis through its elastic centroid.

J_a' = the moment of inertia of the elastic weight of the equivalent arch for the system about the axis conjugate to the vertical axis through the neutral point of the combined arch and pier (about $H_a' = H_a'$ in Fig. 25). Note that the arch becomes unsymmetrical when the pier on one side is combined with it or when the equivalent piers on either side are not alike; or

$J_a' = J_a - \frac{Z_a^2}{I_a}$, in which J_a = the moment of inertia of the axis weight of the equivalent arch about a horizontal axis through its neutral point; Z_a = the product of inertia of the equivalent arch about simultaneous horizontal and vertical axes through its neutral point; and, I_a = the moment of inertia of the elastic weight of the equivalent arch about a vertical axis through its neutral point.

y_p = the vertical distance from the neutral point of the actual pier to the neutral point of the equivalent pier.

y' = the vertical distance from the neutral point of the actual pier to the conjugate axis, $H_a' = H_a'$ (see Fig. 25), or, $y' = y_{np} + x_{np}\frac{Z_a}{I_a}$, in which, y_{np} = the vertical distance from the neutral point of the actual pier to the neutral point of the equivalent arch; x_{np} = the horizontal distance from the neutral point of the equivalent arch to the neutral point of the actual pier; and Z_a' and I_a are the same as previously defined.

W_s = the total elastic weight of the equivalent pier.

W_a = the total elastic weight, $\int \frac{d\,s}{E\,I}$, of the equivalent arch.

W_p = the total elastic weight of the actual pier.

Arch on Equivalent Elastic Piers.—The properties of the arch on equivalent piers may be expressed in terms of I_0, J_0, Z_0, and W_0, in which I_0 and J_0 are the moments of inertia, respectively, about the vertical and horizontal axes through the neutral point of the equivalent arch, of all the elastic weights including the equivalent piers on each side of the span in question; Z_0 is the product of inertia of all the elastic weights about simultaneous horizontal and vertical axes through the neutral point; and W_0 is the total elastic weight of the arch and equivalent piers.

The value of each of these functions may be stated in the following general equations (see Fig. 25):

$$I_0 = I_a + \Sigma\,(W_s\,x^2) - 2\,W_0\,E_x^2 \quad \ldots\ldots(47)$$

$$W_0 = W_a + \Sigma\,W_s \quad \ldots\ldots(48)$$

$$J_0 = J_a + \Sigma\,J_s + \Sigma\,(W_s\,y^2) - W_0\,E_y^2 \quad \ldots\ldots(49)$$

$$Z_0 = \Sigma\,W_s\,x\,y - W_0\,E_x\,E_y \quad \ldots\ldots(50)$$

$$\tan\,\beta = \frac{Z_0}{I_0} \quad \ldots\ldots(51)$$

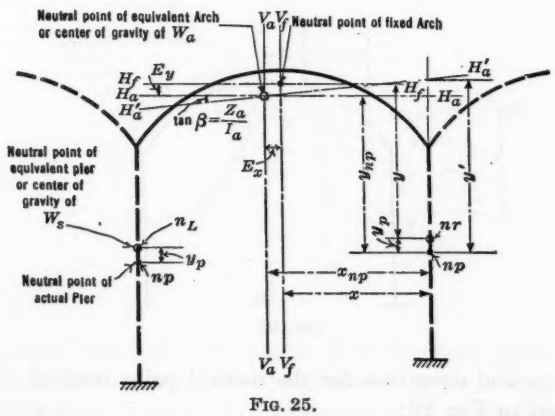

FIG. 25.

As noted in Fig. 25,

E_x and E_y = the co-ordinates of the neutral point of the combined system with respect to the neutral point of the fixed arch as the center of co-ordinates;

β = the angle which the axis conjugate to the vertical axis through the neutral point of the system makes with the horizontal axis through the same neutral point; and,

x and y = the horizontal and vertical distances, respectively, from the neutral points of the equivalent piers to the neutral point of the fixed arch.

The other terms were previously defined.

When the critical arch is the center span of a symmetrical series, the equivalent elastic piers on either side have the same properties, $E_x = 0$, and Equations (47), (49), (50), and (51) simplify to the following:

$$I_0 = I_a + 2\,W_s\,x^2 \quad \ldots\ldots (52)$$

$$J_0 = J_a + 2\,J_s + 2\,W_s\,y^2 - W_0\,E_y^2 \quad \ldots\ldots (53)$$

$$Z_0 = 0 \quad \ldots\ldots (54)$$

$$\tan\beta = 0 \quad \ldots\ldots (55)$$

The influence ordinates for moment, M_0, the horizontal thrust, H_0, and the vertical shear, V_0, at the neutral point of the arch on elastic piers are then determined, as follows, for a symmetrical case (see Fig. 26):

$$M_0 = \frac{W_s\left(\frac{L}{2} - X\right) + \Sigma\, w\, x'}{W_0} \quad \ldots\ldots (56)$$

$$V_0 = \frac{W_s\,\frac{L}{2}\left(\frac{L}{2} - X\right) + \Sigma\, w\, x'\, x}{I_0} \quad \ldots\ldots (57)$$

$$H_0 = \frac{W_s\left(\frac{L}{2} - X\right)y' + \Sigma\, w\, x'\, y}{J_0} \quad \ldots\ldots (58)$$

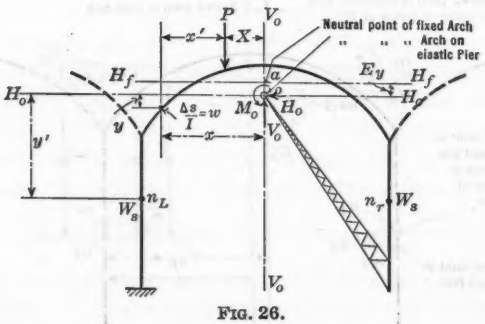

FIG. 26.

These are the typical equations for the neutral point method. The notation used is indicated in Fig. 26:

x = horizontal distance from the neutral point to the elastic weight under consideration.

x' = horizontal distance from the load point to the elastic weight under consideration.

X = horizontal distance from load to the neutral point.

y = vertical distance from the neutral point of the combined system to the elastic weight under consideration (= y' for the pier).

Approximate Method of Analysis.—An approximate method of reduction has been developed which considerably simplifies and shortens the process of computation indicated by Equations (44), (45), and (46) and yet is exact enough for all practical purposes. In Equation (44), $J_a' = J_a$ for the approximate solution. This eliminates the use of the correction factor, $\frac{Z_a^2}{I_a}$, although J_a changes for each equivalent arch. J_a is then computed as the moment of inertia of the elastic weight of the equivalent arch about a horizontal axis through the neutral point of the fixed arch.

In Equation (45), y' is made equal to y. This eliminates the correction factor, $x_{np}\frac{Z_a}{I_a}$. Theoretically y is the distance from the neutral point of the equivalent arch to the neutral point of the actual pier. However, in the approximate method, y is taken as the distance from the neutral point of the fixed arch to the neutral point of the actual pier.

The equations for the approximate analysis can then be re-stated as follows, referring to Fig. 27:

$$\frac{1}{J_s} = \frac{1}{J_p} + \frac{1}{J_a} \quad \ldots\ldots(59)$$

$$y_p = \frac{J_s}{J_a} y \quad \ldots\ldots(60)$$

$$\frac{1}{W_s} = \frac{1}{W_a} + \frac{1}{W_p} + \frac{x^2}{I_a} + \frac{y^2}{J_a + J_p} \quad \ldots\ldots(61)$$

Extended studies have shown that no considerable error results from use of the more convenient formulas.

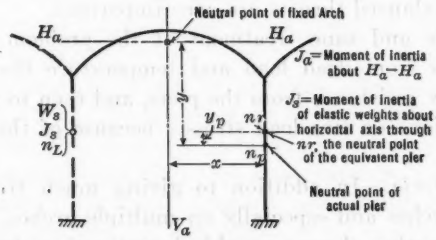

FIG. 27.

Other Considerations in Continuous Arch Studies.—Thus far this discussion has dealt entirely with the solution of a problem in mechanics. The engineering problems involved in the design are much more complicated. Precision in the solution of the problem indicated in mechanics is not very valuable in any particular case unless some estimate can be made of (*a*) the effect of distortions of the pier foundation; (*b*) the factor of safety to be

provided for highly improbable conditions of live load distribution; and (*c*) the effect of the floor system.

Foundation Distortions.—The effect of foundation distortions is not difficult to include if one has any idea of their value; but the prediction of these distortions is one of the most difficult of the unsolved problems of civil engineering. The unlimited variety of foundation conditions for different strata, for strata of different thicknesses, and for the same strata under different conditions of wetness, makes the collection of adequate data almost prohibitive because of the time and money required. Furthermore, the absence of any workable theory as to the laws governing these distortions makes it difficult to use the data for any one stratum. It is not to be supposed that if a pressure of 4 tons per sq. ft. on 100 sq. ft. causes $\frac{1}{4}$ in. of settlement, a pressure of 2 tons per sq. ft. on 100 sq. ft. will cause $\frac{1}{8}$ in. of settlement, and it almost certainly will not be true that a pressure of 4 tons per sq. ft. on 50 sq. ft. will also cause $\frac{1}{4}$ in. of settlement. Existing ideas are so vague as to have practically no quantitative value whatever; but in a continuous series of arches the matter is very important. What might seem a slight rotation at the base of a pier may produce at the pier top a displacement several times as great as the elastic distortion of the pier. Preliminary reports on the work of W. M. Wilson, M. Am. Soc. C. E., at the University of Illinois, and of Clyde T. Morris, M. Am. Soc. C. E., at Ohio State University, indicate that this does occur, but their measurements were on the effect of loads of considerable duration—dead load and temperature. The effect of transient live load would not be so great.

In a continuous series of arches, economy based on the topographic conditions and æsthetic considerations usually indicates that the span lengths shall not be equal, but shall decrease toward the abutments. In such cases if the dead load thrusts are balanced at the pier heads, the temperature thrusts are unbalanced and *vice versa,* and if the thrusts are balanced, the moments at the pier heads are unbalanced. The unbalancing of the moments is not very important, but unbalanced thrusts are very important.

Probably a safe and sane treatment of the problem is to equalize any unbalanced thrusts from dead load and temperature throughout the series, thus neglecting any resistance from the piers, and then to neglect the effect of foundation distortions on live load stresses because of the transient effect of live load.

Live Load Criteria.—In addition to giving much time to the study of existing data on arches and especially on multiple arches, the writer has had the pleasure of directing three very able investigations by graduate students in this field. In 1923-24, Mr. Lorenz Straub made an extensive study along the lines indicated previously for exact analysis, determining the influence lines and maximum stresses for different pier heights. In 1924-25, Mr. H. E Wessman extended the work previously done by Mr. Straub and made studies of the probability of the indicated live load conditions and the effect of these conditions on the design. This investigation resulted in a set of specifications for computing live load maxima, together with a simplified procedure of

computation of which the approximate method indicated is a part. Perhaps the most interesting part of the work was in showing the heavy burden which could be put on the design by a too literal interpretation of influence lines. Thus, under certain conditions, the pier would be required to carry as a cantilever beam unsupported by adjacent spans nearly all the full live load thrust of one span. It is easy to see that this is a tremendous burden, but it can be shown that the probability of the required congested loading is small. There is, however, an unfortunate tendency in the literature of indeterminate structures to assume that the construction of influence lines gives a complete solution of design problems, whereas interpretation of these graphs is a much more difficult problem, and an influence line not properly interpreted may become an expensive luxury to a client.

Economic Proportions.—In 1925-26, Mr. E. A. MacLean (now Associate Professor of Civil Engineering at the Agricultural and Mechanical College of Texas) took up the study of the economic proportions of continuous arch series, investigating twelve different combinations of pier proportion and rise ratio. This problem, the ultimate aim of structural studies, is one of great complexity. Yet, Mr. MacLean was able to lay down certain tentative principles to guide the designer. That the rise ratio should increase with the pier slenderness was, of course, indicated from the beginning.

Influence of the Floor.—At present the writer has in hand extensive studies the first object of which is to determine the desirability of considering the effect of the integral action of the floor and arch ring in fixed arches and especially the effect of expansion joints in the floor. Later, it is intended to study the influence of the floor in continuous arches. In the case of fixed arches, the question of considering the floor is apparently one of economy, because (except in some cases for the effect of improperly located expansion joints on temperature stresses) no danger is to be expected from neglecting it. In a continuous arch series, on the other hand, where the spandrel posts are stiff, neglect of the floor invalidates the whole computation. The controlling element in the analysis of the continuous arch problem is the distribution of unbalanced live load thrusts at the pier head between the pier and the two connecting arches in proportion to their resistance to horizontal displacement. It is evident that the resistance of an arch to horizontal displacement (change of span) is decidedly affected by the interaction of arch ring and floor system.

Difficulty of Correlation of Data.—A continuous arch series offers possibilities of economy and graceful beauty which are unusually attractive to bridge engineers. The problems involved, however, are complicated, their study requires both time and patience, and a synthesis of the results is not to be made hastily. Many existing series are neither economical nor beautiful, and past practice cannot here be considered to be a good guide to future performance. It is, therefore, of the greatest importance that the principles be laid down after full and careful discussion by those who have studied the problem intensely and not by hasty dicta or arbitrary dogmas.

DISTRIBUTION OF REINFORCING STEEL IN CONCRETE BEAMS AND SLABS

Discussion*

By John J. Gould, Assoc. M. Am. Soc. C. E.

John J. Gould,† Assoc. M. Am. Soc. C. E. (by letter).‡—The author has shown certain methods of designing reinforced concrete beams and slabs. He has failed, however, to substantiate these methods by actual tests on beams, or by scientific reasoning; nor does he justify in any way his rejection of the Theorem of Three Moments in designing reinforced concrete structures.

Generally speaking, the author believes that by placing reinforcing steel in a certain way he is able to regulate exterior moments and working stresses; that the distribution of the steel—a part to the center and the remainder to the support, or *vice versa*—is entirely optional with the designer.

In Fig. 20 is shown what the deflection line of a continuous beam would be if all the steel were placed in the top and the continuous beam were designed as a series of cantilevers. Besides the questionable stability of the system, cracks in the middle of the spans would occur. Fig. 20 leaves no doubt about the author's erroneous fundamental assumptions in designing reinforced concrete structures.

According to the author's "General Principles", T-beams "cannot readily be analyzed by the three-moment theorem or similar theory".§ The writer believes that the extensive tests of continuous reinforced concrete beams, made in Germany by Dr. E. Probst‖ and other engineers, may dissipate some of the existing doubt as to whether or not the Theorem of Three Moments should be applied to reinforced concrete structures

These tests of T-beam sections show how little influence all errors make on exterior moments. They were made under the assumptions of constant moment of inertia, straight line variation of concrete stresses, and neglect of shearing stresses.

In all these tests the theoretical moments of continuous beams were calculated on the basis of the Theorem of Three Moments. These moments were then compared with the moments calculated from measured deformations of the concrete and steel after the beams had failed from ultimate loads. The sections of all beams were of the same shape and dimensions. The reinforcing bars were located according to the theoretical moments and shears.

Test Type I, of T-beams on two supports, 3.0-m. span, was made for comparison with other continuous beams. In Test Type II, of continuous T-beams on three supports with two equal 3.0-m. spans, the failures occurred over

* Discussion on the paper by Boyd S. Myers, M. Am. Soc. C. E., continued from February, 1927, *Proceedings*.

† Structural Engr., L. H. Nishkian, San Francisco, Calif.

‡ Received by the Secretary, January 3, 1927.

§ *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1107.

‖ "Vorlesungen über Eisenbeton," Bd. I, Berlin, 1917.

the interior supports, which indicates that the maximum moment of the beam is negative. The difference of 20% between the theoretical moment over the interior support and the moment calculated from actual stresses is because the loads were not uniformly distributed and a possible settlement of the support may have occurred.

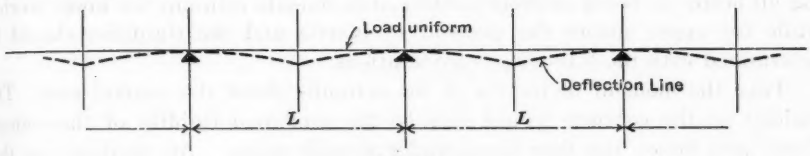

FIG. 20.

The comparison of relative deflections between Test Types I and II in the center of the beams gave a theoretical relationship, $R\,e$, of,

$$R\,e = \frac{D\,e\ \text{II}}{D\,e\ \text{I}} - \frac{P\,L^3}{192\,E\,I} \div \frac{5\,P\,L^3}{384\,E\,I} = 0.40$$

in which, $D\,e$ I is the deflection of the beam, Test Type I; similarly, for $D\,e$ II, $D\,e$ III, etc. By comparison the actual measured deflections show the following relationship:

	Before cracks occurred.	After the first cracks occurred.	At approximately one-half the ultimate load.
Loads, in tons per meter =	3.6	6.6	10.4
$R\,e = \frac{D\,e\,\text{II}}{D\,e\,\text{I}}$ =	0.90	0.42	0.45

In Test Type III, a continuous T-beam on four supports, three equal, 3.0-m. spans, the theoretical moments show very close relationship with the moments calculated from measured deformations.

The comparison of relative deflection between Test Types I and III for the end spans (the middle span was not loaded) gives the following relationship:

$$R\,e = \frac{D\,e\ \text{III}}{D\,e\ \text{I}} = 0.260\,\frac{P}{E\,I} \div \frac{5\,P\,L^3}{384\,E\,I} = 0.74$$

while the comparison of the actual measured deflections show a relationship of:

Loads, in tons per meter =	3.6	6.6	10.4
$R\,e = \frac{D\,e\ \text{III}}{D\,e\ \text{I}}$ =	0.69	0.63	0.69

In Test Type IV, the construction is similar to Test Type III. The beams, however, were built integral with the supports, that is, a typical reinforced concrete structure. The deflections were found to be smaller than those in Test Type III. The influence of the columns on the beam moments were about 20% $\left(\text{for Test Type III, } M = \frac{1}{9.7}\ W\,L\text{; and for Test Type IV, } M = \frac{1}{12}\ W\,L\right)$.

The measured relative deflections between Test Types I and IV are:

Loads, in tons per meter =	3.6	6.6	10.4
$R\,e = \frac{D\,e\ \text{IV}}{D\,e\ \text{I}}$ =	0.36	0.37	0.34

The comparison between these coefficients and the coefficients found for Test Type III shows that the columns fix the beams to a considerable degree. If the beams were entirely restrained at their ends, these coefficients would become 0.20.

In an ordinary reinforced structure the writer would recommend considering all beams as being entirely restrained at outside columns for lower stories, while for upper stories the moment of inertia and the rigidities should be determined with the following considerations.

Take the moment of inertia of the columns about the neutral axis. The rigidity of the columns would then be the combined rigidity of the column above and below the floor-beam under consideration. An analysis on this basis would result in a bending moment for the beam at the outside support with an average value of $\frac{W L}{12}$ to $\frac{W L}{16}$.

Test Type V, was a continuous beam on six supports, with spans of 3.0, 2.5, 4.0, 2.5, and 3.0-m., the 3.0 and 4.0-m. spans only being loaded. The comparison of relative measured deflections between Test Types I and V in the center of the 3.0-m. spans, gives a relationship of:

Loads, in tons per meter =	3.6	6.6	10.4
$R\,e = \frac{D\,e\,\mathrm{V}}{D\,e\,\mathrm{I}} =$	0.80	0.67	0.72

These coefficients are very similar to those found in Test Type III. The theoretical exterior moments compare closely with the moments calculated from actual stresses. The continuous effect of five spans, therefore, should be considered.

Conclusions.—Dr. Probst's tests of continuous beams have shown that:

(1) Moments computed on the basis of the Theorem of Three Moments check within a few percent. with the moments calculated from actual stresses.

(2) The measured relative deflections of a simply supported reinforced concrete T-beam and continuous T-beams are almost identical with the theoretical relative deflections of beams of homogeneous material and of constant moment of inertia.

(3) A few small cracks occurred at the points of zero moments, thereby locating these points at positions agreeing with the Theorem of Three Moments.

(4) In curvature, the deflection line of reinforced concrete T-beams resembles that of beams of homogeneous material and of constant moment of inertia.

(5) Continuous reinforced concrete beams, therefore, should be designed in accordance with the laws of the Theorem of Three Moments.

(6) The designer should take into consideration the end restraint of beams which frame into columns.

(7) Arbitrary coefficients for computing reinforced concrete structures are irrational, and in very many cases contradict the Theorem of Three Moments.

GRIT CHAMBER PRACTICE

A SYMPOSIUM

Discussion*

By R. S. Rankin, Esq.

R. S. Rankin,† Esq. (by letter).‡—The design of the grit chamber for Decatur, Ill., possesses certain peculiarities uncommon to others, that may be of interest. The chamber is located at the end of a 60-in. intercepting sewer about 14 000 ft. long. Beyond the grit chamber are the Venturi meter, Imhoff tanks, and the remainder of the treatment works. Between the grit chamber and the Venturi meter, area and sufficient head have been provided for future sewage treatment developments. As a result of this provision

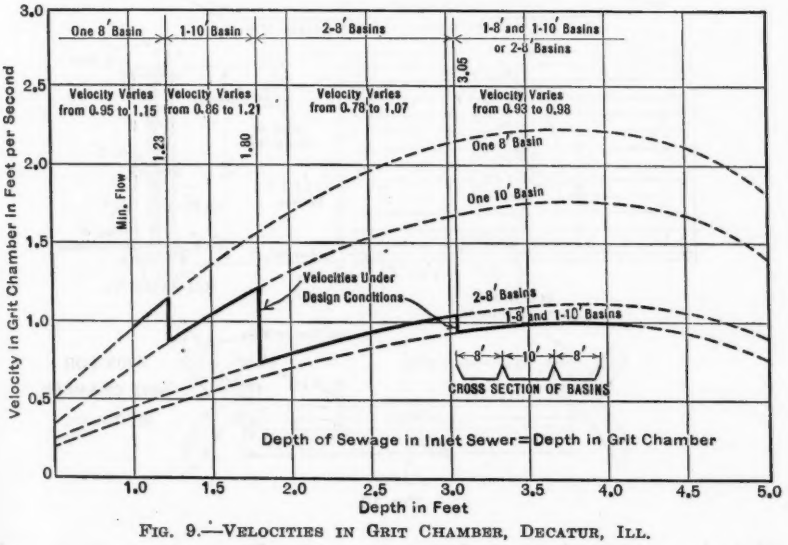

Fig. 9.—Velocities in Grit Chamber, Decatur, Ill.

there existed between the water levels in the grit chamber and in the Imhoff tanks, a difference in head, which varied for different quantities of sewage flow. The grit chamber design was based on the assumption that there would be a definite water level for any given flow through it with the water level

* Discussion on the Symposium on Grit Chamber Practice, continued from December, 1926, *Proceedings*.

† With the Dorr Company, Inc., New York, N. Y.

‡ Received by the Secretary, December 31, 1926.

at the outlet of the chamber lower by the amount which had been allowed for future devices. This difference in head is now controlled so as to produce a uniform velocity in the chamber.

In the design of the grit chamber, two major assumptions were made: (1) That the water level in the chamber would be the same as that in the incoming sewer; that is, the water level would rise and fall the same as in the sewer; and (2) that not more than two of the three "flowing through" compartments in the chamber would ever be used at one time.

The quantity and the level of the flow in the incoming sewer, which is 60 in. in diameter, varies according to the usual curve of a sewer having a circular cross-section. It was decided that an optimum velocity of 1 ft. per sec. would be desirable in the grit chamber. The theoretical cross-section that would produce this required velocity was computed for various levels. It was slightly modified to straighten out the irregularities to a section which would be readily constructed.

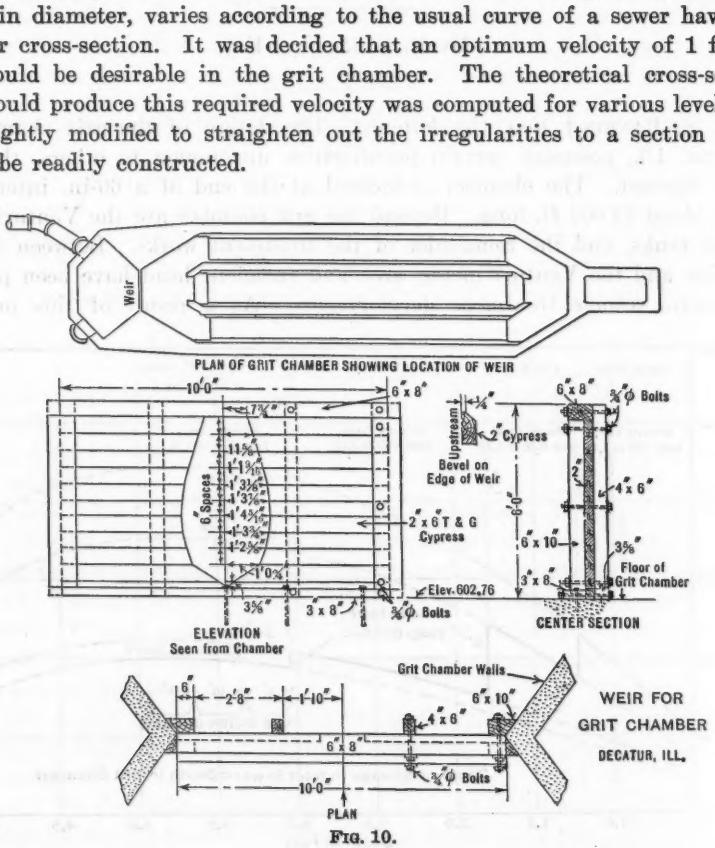

FIG. 10.

With the water level at any elevation corresponding to that in the incoming sewer, the theoretical flow and velocity in the grit chamber will be very close to 1 ft. per sec., as shown in Fig. 9.

In order to use up the head at the outlet and still maintain the correct level in the grit chamber, it was necessary to design a rather peculiar weir. This weir is of the submerged type designed on the basis that at any level there would be a definite head available for discharge. After several cut-and-try computations, a theoretical profile of the weir was determined and a section

approaching this as nearly as practicable was adopted. The weir with this profile was constructed of wood and installed in the outlet as shown in Fig. 10, with the idea that modifications might have to be made. However, no modifications in the weir have become necessary, as current meter measurements in the grit chamber from time to time have shown that the velocities are very close to 1 ft. per sec.

The drainage of the grit chamber is accomplished by under-drains placed in a recess along the sides and near the bottom of each "flowing through" compartment, and consist of gravel-packed drainage pipes laid with open joints. The gravel is held in place by cast-iron gratings set flush with the side-wall of the chamber.

For removing the grit from the basin, each compartment has been provided with three outlets of 18-in. cast-iron pipe with a gate-valve control. The grit chamber is set high enough above the ground so that dump cars may run beneath on industrial tracks and the contents of any chamber may be discharged through these outlets directly into the cars.

The control of the flow is accomplished by means of wooden stop planks at either end and a swinging wooden gate at the inlet end. The operation of the grit chamber in general has worked out satisfactorily and conforms substantially with the design.

PRODUCING CONCRETE OF UNIFORM QUALITY

Discussion*

By John G. Ahlers, M. Am. Soc. C. E.

John G. Ahlers,† M. Am. Soc. C. E. (by letter).‡—The wealth of experience shown in Mr. Young's paper is very impressive and adds greatly to the conviction of his conclusions.

As he states the entire problem of producing concrete of uniform quality is really an economic one and resolves itself into, first, arriving at the proper water-cement ratio for any required strength; and, second, combining the materials available so as to give the highest yield with the selected water-cement ratio.

Like many others, Mr. Young forgets that strength can be used as a criterion of all the other desirable qualities of concrete. Contrary to his opinion, the writer believes that the design or mixture should be based on strength alone. If a high strength is obtained, all the other desirable qualities will follow—as proven by Professor Abrams§ and as shown by Professor M. O. Withey, of the University of Wisconsin, in connection with permeability. If plasticity and workability are made the limiting factors then the measurement, fineness modulus, and volume of fine to coarse do not require exact control, and the whole problem of producing concrete of uniform quality becomes very simple.

It is particularly interesting to note that in the large volume of concrete produced under Mr. Young's supervision, he finds that the method of handling material prior to measurement is not of great effect on the quality of the concrete, except that any method which allows segregation of the aggregate makes it more difficult to obtain uniform concrete. The writer's experience is that concrete of uniform strength is produced even if there may be segregation, but the yield or economy of the different batches may vary. In other words, in a poorly graded batch not as large a volume of concrete will be produced as in a batch with the fine and coarse aggregate well graded.

The writer rather questions the statement that the measurement of cement is not a problem of control.‖ It is a known fact that the different brands of cement will have different weights in the bags and that within the same brand there will be variations from bag to bag. Also, there are many bags

* Discussion on the paper by Roderick B. Young, Esq., continued from February, 1927, *Proceedings*.

† Secy.-Treas., Barney-Ahlers Constr. Corporation, New York, N. Y.

‡ Received by the Secretary, January 19, 1927.

§ "Bond Between Steel and Concrete."

‖ *Proceedings*, Am. Soc. C. E., September, 1926, Papers and Discussions, p. 1396.

broken on every concrete operation, and many bags with lumps. It is not often that the bags are shaken in the same manner each time. It, therefore, becomes very important to watch the quantity of cement going into every batch. A system that allows simple supervision of this quantity will simplify the problem of control. A system that uses the cement as a measure of the water makes this control absolute and allows no variation regardless of the irregularities in the measurement of cement.

Regarding the moisture content* Mr. Young's experience parallels that of the writer to the effect that no method entirely limits the necessity of determining the moisture contents of the aggregate. It is extremely important that moisture be determined in both coarse and fine aggregate and allowance made for the average maximum condition. By so doing the concrete will be slightly stronger than is expected or required.

As to the importance of the period of mixing,† if the testing of the concrete produced under job conditions is made the standard from which other strengths may be obtained, the period of mixing does not enter into the problem of field control. It is a fact, however, that contractors may obtain higher strengths and, therefore, concrete at lower cost if they allow a longer mixing time when producing concrete for a given strength.

The paper is very helpful in further developing the general knowledge of field control of concrete. The system of field control is becoming more standardized and simplified each year. On field work undertaken by the writer's organization it has reduced to the following simple rules:

(1) Under job conditions manufacture concrete with a fixed water-cement ratio. This is called the "job constant of strength" and fixes the "job curve".‡
(2) Test this concrete at 7 days and interpret the test results for 28 days.
(3) From these test results determine what other water-cement ratios will give the strengths required on the job.
(4) Use a mechanical device for giving these water-cement ratios and add as much as possible well-graded fine and coarse aggregate to the water-cement mixture.
(5) Check the resulting concrete from time to time against the "job constant of strength".

* *Proceedings*, Am. Soc. C. E., September, 1926, Papers and Discussions, p. 1397.
† *Loc. cit.*, p. 1398.
‡ *Loc. cit.*, August, 1926, Papers and Discussions, p. 1269.

WATER-RATIO SPECIFICATION FOR CONCRETE

Discussion*

By John G. Ahlers, M. Am. Soc. C. E.

John G. Ahlers,† M. Am. Soc. C. E. (by letter).‡—The general criticism of this paper would be that the specifications are too strict in not allowing enough latitude for the contractor who will honestly endeavor to make concrete of a given strength so that he may benefit from the resulting economies. Particularly is it a handicap where the water ratios are restricted to the maximum permissible, if the strengths are produced uniformly and within limited tolerances. A contractor should be permitted to change the water-cement ratio and obtain the benefit from his supervision and control methods.

The specification is correct in allowing the proportions of aggregate to cement to be governed only by the requirements of workability, but it should go further and allow a change in the water-cement ratio where proper careful control methods are used to keep this strength.

The specification brings out the point that has been stressed very much in the writer's own practical contracting experience that the object of owner and contractor can be made identical and the use of the water-ratio specification is the greatest step forward in harmony between the Contracting and Engineering Professions in practical field work. This has been particularly stressed by the writer,§ based on actual experiences on twenty building operations where water-ratio control was used on all concrete placed in the structures.

It has also been the writer's experience that there is no difficulty in estimating the quantities of material used under control by the water-cement ratio. After one or two operations every contracting organization will arrive at its own constants of cement factors for different strengths and for different types of materials used.

Collection of these data alone is of great economic value to the contractor and ultimately to the engineer, architect, and owner, because when these economies are realized the contracting field will be able to produce better concrete at lower prices than in the past.

For water-ratio control the ordinary type of tank is not satisfactory. The special equipment used by the writer has previously been described.§ No

* Discussion on the paper by F. R. McMillan, M. Am. Soc. C. E., and Stanton Walker, Assoc. M. Am. Soc. C. E., continued from January, 1927, *Proceedings*.

† Secy.-Treas., Barney-Ahlers Constr. Corporation, New York, N. Y.

‡ Received by the Secretary, January 19, 1927.

§ *Proceedings*, Am. Soc. C. E., August, 1926, Papers and Discussions, p. 1268.

inconvenience has been caused by the necessity of correcting the quantity of mixing water for the moisture in the aggregate.

Perhaps the chief value of water-ratio control lies in that both contractors and engineers are made to study the available cement and aggregate more thoroughly and thus to get a different perspective on the factors entering into the production of good concrete. The following improvements in the specification are suggested:

(*a*) Instead of specifying the maximum permissible water-cement ratio, exact water-cement ratios should be specified, based on actual tests of concrete produced under job conditions.

(*b*) Portland cement should be tested for compression as well as for tension as this would give a definite indication of its desirability and of the strength that may be expected for certain water-cement ratios. A specification for tests of Portland cement would be a good indicator of the water-cement ratios to be specified for the work. Then the time of mixing need not be limited; this factor is already covered.

UNIT STRESSES IN STRUCTURAL MATERIALS

A SYMPOSIUM

Discussion*

By Clement E. Chase, M. Am. Soc. C. E.

Clement E. Chase,† M. Am. Soc. C. E.—Now that the work of the Special Committee on Stresses in Structural Steel is finished and its differences of opinion lie partly revealed and partly concealed in the wording of the Majority and Minority Reports,‡ there may be a feeling that enough has been said by the members of the Committee. On the other hand, an informal discussion such as this, and more particularly the contribution from the steel makers' point of view by Mr. Kenney,§ makes this an opportune occasion to present a few of the considerations which influenced some of the members of the Committee to oppose increasing the basic working stress to more than 18 000 lb. per sq. in.

Those who recall the Progress Report‖ of the Committee on Stresses in Structural Steel presented at the Annual Meeting of the Society in 1924 will remember that the Committee then apparently understood that as a foundation for its mission of studying the problem of the "stresses in structural steel" it was necessary first to make a survey of the material, "commercial structural steel", as it is made and furnished. That progress report summarized the study the Committee had made of mill records of the routine acceptance tests of various classes and thicknesses of structural steel. The speaker was Chairman of the Sub-Committee that went to the steel mills to gather those records and believes them to have been truly representative of mill tests as now made—with the characteristic variation in results at different mills which the experienced inspector has learned to expect.

It was part of the Sub-Committee's program, approved at the time by the Committee, to follow this preliminary survey with a series of tests on specimens cut from identical pieces of structural material. One specimen from each piece was to be put through the regular routine of testing at the steel mills and its companion specimen was to be subjected to precise laboratory testing to determine the character of the stress strain curve, in addition to checking properties revealed by the mill tests. It was thought that such

* Discussion on the Symposium on Unit Stresses in Structural Materials, continued from February, 1927, *Proceedings*.

† Prin. Asst. Engr., Delaware River Bridge Joint Comm., Philadelphia, Pa.

‡ *Proceedings*, Am. Soc. C. E., March, 1925, Papers and Discussions, p. 392; March, 1926, Papers and Discussions, p. 439.

§ *Loc. cit.*, October, 1926, Papers and Discussions, p. 1734.

‖ *Loc. cit.*, March, 1924, Society Affairs, p. 263.

a comparative study would afford a real basis of knowledge of the material involved, to serve as a starting point for an estimate of its action in full-scale members and structures.

That program was never carried out. The Committee subsequently decided that there was sufficient information already available to its members, when taken in conjunction with this preliminary survey of mill test results, fully to define structural steel as made to-day and to enable the determination of revised, up-to-date, working stresses. Much reliance was placed on the average mill test results, although in the final analysis about 6 000 or 7 000 lb. per sq. in. was "chopped off" the averages for good measure. It was assumed that a yield point of 30 000 lb. per sq. in. could be relied on, and the increased working stresses were based on this value. There appeared to be a feeling that this was a very conservative figure.

Those who have closely observed the product of the steel mill, as representatives of the purchasers, agree that as to uniformity of chemical content and ultimate strength the present-day product is highly satisfactory. As regards yield point or elastic properties it is another story. The experienced observer knows that there is no high degree of uniformity of elastic properties in the actual product and that the values for yield point reported in mill tests are not an accurate index of the quality of the steel in this regard. It is apparent from the discussion that at least one steel man of wide experience does not consider that the ordinary mill tests give information as to yield point on which any great degree of reliance should be placed. That he does not consider the yield point an important property even when carefully determined is another matter. The structural engineer dealing with static structures is thoroughly convinced of the basic importance of this property in his work. The mechanical engineer is generally interested in fatigue-resisting properties. He may conclude from recent tests that fatigue resistance can better be judged from the ultimate strength of the material than from its elastic limit or yield point.

The speaker's belief is that a series of stress-strain curves taken from fully representative samples of structural steel of moderate thickness would show the ratio of yield point to range from 55% down to 50% of the ultimate strength in a fair proportion of tests and even less in some cases—for material with an ultimate strength near the lower limit of the specifications (55 000 lb. per sq. in.) this means a real yield point strength less than 30 000 lb. per sq. in.

The report of the Society's Special Committee on Steel Columns and Struts* shows a stress-strain curve for material from the root of an 8-in., 18-lb., I-beam. This has a "U. L. P." of 27 200 lb. per sq. in., a yield point (taken as 0.1% set) of 29 000 lb. per sq. in., and an ultimate strength of 58 900 lb. per sq. in. Had the ultimate strength been 55 000 lb. per sq. in., a proportionate decrease in yield point could be expected. Marburg† reports for a standard 24-in., 80-lb., I-beam, the values, in pounds per square inch, given in Table 1.

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), Plate XXXV, p. 1631.

† "Tests of Steel Beams," *Proceedings*, Am. Soc. for Testing Materials, 1909, p. 385.

Marburg found the elastic limit and yield point by extensometer. His values for elastic limit are really "proportional limits", the difference being too slight a degree of inelasticity to be of practical moment, but his values for yield point are pertinent. Note that the ratios of yield point to ultimate strength are, respectively, 55, 51, and 46 per cent.

TABLE 1.

	Elastic limit.	Yield Point.	Ultimate strength.
Web	24 500	30 500	54 800
Flange	25 400	28 400	55 000
Root	15 200	28 100	60 100

The falling off in the ratio of yield point to ultimate strength will be even more marked as the thickness of material is increased. Tests made at Drexel Institute* showed the average yield point of specimens cut from 12-in., universal mill-plates from one heat and rolled to various thicknesses, as shown in Table 2.

TABLE 2.

Thickness, in inches.	Average yield point, in pounds per square inch.	Average ultimate strength, in pounds per square inch.	Ratio of yield point to ultimate strength.
0.355	38 800	63 720	60.5
0.475	36 880	63 770	57.8
0.755	33 420	62 150	53.8
1.000	27 380	60 850	44.9
0.995	25 700	58 600	43.8

The Majority Report of the Special Committee on Stresses in Structural Steel passed lightly over the question of thickness of material. To the speaker this has seemed a point that can not be slighted in considering working stresses; for, after all, the variations of yield point in steel of the same ultimate strength are largely due to whether the material is thick or thin. It is not considered a practical mill procedure at present to change the rolling temperature and other conditions so as to bring out a uniform "elastic ratio", that is, a uniform ratio of yield point to ultimate strength. As regards the thick material, the Majority Report merely states that for buildings all material is to be subject to the specifications of the American Society for Testing Materials, but that Bessemer steel is to be barred.

It further states that if he is using heavy material, the engineer should assure himself that all the material meets the requirements of the specifications. Any one who has had experience with testing materials under the standard specifications will know how little such an injunction takes account of actual conditions. The ordinary mill acceptance test for even the thickest flange beam or column is cut from the web. Any engineer would find it an

* *Transactions*, Am. Soc. C. E., Vol. LXIII (1909), p. 264.

interesting experience to go to the mill and attempt to get his test pieces cut from the thick part of the section. He would certainly be told that his contract called for steel to be tested and accepted under standard conditions, and that no one had ever heard of an acceptance test specimen being cut anywhere but from the web.

There is an increasing tendency on the part of structural engineers to use thick slabs, plates, and sections, which makes it highly important that the actual physical characteristics of such material be fully known. Thick material has such obvious advantages in many cases that its use will doubtless increase.

At present there is being undertaken by a committee under the auspices of the American Society for Testing Materials, an investigation of the yield point and other characteristics of structural material. It is hoped that this committee will be successful in making a true and accurate survey of the elastic properties of the materials that are so confidently used in building construction of ever-increasing size and complexity.

Even when a careful survey of the properties of specimens of structural steel is available only a start will have been made to acquire the knowledge of steel structures that many assume is already attained. Working stresses cannot be based on yield points of specimens of the material, unless the relation between the yield points of the specimen and of the full-sized member is known. Many have taken for granted, apparently, that there is no difference between the two, at least when considering tension members.

A study of the meager data on tension members, other than eye-bars, gives ample warning that working members may not show as high yield point strength as specimen tests. The speaker has analyzed the tension tests made for the Board of Engineers of the Quebec Bridge* and by J. E. Greiner, M. Am. Soc. C. E.† A summary of the results is given in Table 3. The tension tests made by McKibben do not report the yield point strength of the members.

Examination of the Quebec and Greiner tests indicates that it would not be safe to count on greater yield point strength for the actual structure than 47% of the ultimate strength of the specimen material. For the minimum permissible ultimate strength of 55 000 lb. per sq. in., this gives approximately 26 000 lb. per sq. in.; and the fact that a single, 2-lb. specimen tested to represent a 50-ton heat of structural steel may show an ultimate strength of 55 000 lb. per sq. in. is no assurance that all the other material in the heat has even that strength. It is a well-established fact that steel rolled from a single heat will vary 5 000 to 10 000 lb. in ultimate strength, and the test specimen may not happen to be cut from the weakest material. The fact that most of the members of a structure contain material well in excess of an ultimate strength of 55 000 lb. per sq. in. will lend no additional strength to those members that are weaker.

It is frequently assumed that the tensile working stress is not an important one as tension members do not play a large part in building construction. However, one of the most vital elements in many large buildings at the present

* Quebec Bridge Report, Appendices XI–XII.

† *Transactions*, Am. Soc. C. E., Vol. XXXVIII (1897), p. 41.

TABLE 3.—FULL-SIZED TENSION TESTS.*

QUEBEC BRIDGE TESTS.

Description of test.	MATERIALS.		Ultimate strength of net section, in pounds per square inch.	Percentage of elongation in 200 in.	Elastic limit; net section, in pounds per square inch.	Yield point of net section, in pounds per square inch.	Ratio of yield point to ultimate strength, in pounds per square inch of materials.	Number of tests averaged.
	Yield point, in pounds per square inch.	Ultimate strength, in pounds per square inch.						
One plate, 36 in. by ⅝ in. by 26 ft. 8 in...	41 420	68 700	55 352	2.45	29 000	32 710	47.6	10
One plate, 36 in. by ⅝ in. by 26 ft. 8 in...	40 165	63 170	52 170	4.5	24 000	28 350	44.8	2
							47.1	12
Three plates, 36 in. by 5/16 in. by 26 ft. 8 in.	44 080	65 620	53 860	3.9	24 230	30 400	46.3	6
Three plates, 36 in. by 5/16 in. by 26 ft. 8 in	43 430	62 910	66 760	3.9	29 900	35 400	56.3	4
							50.3	10
Two plates, 24 in. by ½ in. Four angles, 4 by 3 in. by 5/16 in.	46 700	63 620	56 500	....	27 000	33 700	53.0	2
Two plates, 24 in. by ⅜ in. Eight angles, 4 by 3 in. by 5/16 in.	45 630	63 550	57 700	....	26 100	32 600	51.3	2
One plate, 36 in. by 5/16 in. Two plates, 28 in. by 5/16 in. Four angles, 4 by 3 in. by 5/16 in.	44 660	63 770	56 690	....	28 900	31 640	49.6	2

* Values for yield point for full-sized tests are the stresses at which the extensometers began to creep perceptibly under a steady load.

TABLE 3.—(*Continued*).

GREINER TESTS.*

Description of tests.	SPECIMEN TESTS.		FULL-SIZED TESTS.		Ratio of yield point to ultimate strength of material.
	Elastic limit, in pounds per square inch.	Ultimate strength, in pounds per square inch.	Elastic limit, in pounds per square inch.	Ultimate strength, in pounds per square inch.	
Built-up tension members, Nos. 3-10, Series A; Nos. 11-18, Series B; average of sixteen tests of full-sized members, metal, ¼ in. thick	38 250	58 200	44 130	50 650	75.8
Single angle tension members connected by both legs; average of two tests, metal, ⅜ in. thick	38 850	59 350	26 633	52 261	44.9
	36 800	55 200	28 247	49 838	51.2
					Average....48.0
Members connected by one leg	38 850	59 350	22 832	44 392	38.5
	36 300	55 200	24 183	42 042	43.8
					Average....41.2

* Elastic limit determined by scaling of specimen, that is, what would now be called yield point.

time are the heavy trusses which form a part of the lower construction and carry the load of as many tiers as the architect or designer desires to put above them. In those trusses one must admit that the tension member occupies as vital a place as it does in any railroad bridge; and upon what full-sized tests are the designs of such members and trusses based?

The speaker feels that there is a reason for the designer of steel structures to be rather humble in the presence of those of his colleagues who specialize in concrete and have for guidance the many tests in reinforced concrete members and structures. The steel designer must admit that hundreds of thousands of tons of steel are used annually in steel buildings with practically no information at all from tests as to the strength of the completed structures or many of the details and members used so freely in their construction. The little information that exists on the strength of large sized steel members seems to indicate need of caution before making large inroads into the unknown margin of safety. In 1880 Bouscaren spoke of the "irrepressible 'factor of safety' invented to cover two large fields, the 'incidental' and the 'unknown', the limits of which should gradually contract under the light of well observed facts." That they have contracted is undeniable. For the designer of buildings to assume that there is no longer any element of the "unknown" in his materials and structures is again to invite the disasters that in the past have served to chasten the designer of bridges.

WATER-PROOF MASONRY DAMS

Discussion*

By Messrs. Thomas H. Wiggin, Jerome W. Howe, W. H. R. Nimmo, and E. W. Lane

Thomas H. Wiggin,† M. Am. Soc. C. E.—The author develops an idea that is worthy of careful examination. For a great many years the importance of excluding water pressure as far as possible from joints in and beneath gravity dams has been realized by designers; but hitherto the preventive steps taken in construction have been (*a*) to grout seams in the foundation; (*b*) to carry a cut-off wall down in a trench below the general level of the dam; (*c*) to strive for dense masonry with joints filled as completely as practicable; and, in a few cases, (*d*) to provide drainage near the up-stream face of the dam.

In repair work after excessive seepage has occurred, various schemes involving caulking, gunite, and an additional layer of masonry, have been used to try to correct defects. A recent case of this kind is the Delta Dam at Rome, N. Y., a part of the water supply system of the Barge Canal. Here a covering of rich concrete masonry from 1 ft. to 4 ft. thick has been used under specifications with which the speaker was informally concerned. This type of construction was adopted after considering methods similar to those suggested by the author.

Men of two schools of thought are designing dams, those of one school assuming that provisions like those already listed will be successful in preventing uplift water pressure and hence adding no weight in the design on account of uplift; and those of the other school adding weight as well as taking the precautions to reduce uplift pressure. It is impossible to know what proportion of dams has the extra weight to provide for uplift pressure (on some assumption) and what proportion has not. In general, the older dams lack the provision and some of the recent dams follow the older method, especially where economy is thought to be important. Success of the older dams would be justification for the lighter designs but for the unfortunate fact that a comparatively large number of gravity dams have failed, although the causes are not usually easily analyzed. Beyond doubt, however, sheer additional weight would have helped them.

For several years the speaker has urged that the Society undertake energetically to resolve these doubts as to what extent uplift pressure is a real factor. Some work on this is being done by one of the nine Sub-Committees of the Society's active Special Committee on Irrigation Hydraulics, as well as

* Discussion on the paper by W. Watters Pagon, M. Am. Soc. C. E., continued from February, 1927, *Proceedings*.

† Cons. Engr., New York, N. Y.

by the U. S. Department of Agriculture; but the matter seems worthy of a special committee with means to enlist systematic co-operation from every quarter.

Ice pressure is a kindred and very important matter which could well be included in this investigation, since here, again, two schools of design are operating and important differences in weight of structures result.

Assuming it to be practicable, by use of water-proofing such as the author proposes, to eliminate all doubt as to uplift pressure in the masonry, there still remains the more difficult questions of junction between the dam and its foundation as well as of joints in the rock immediately beneath the dam. Saving in weight in the dam would be impossible unless these lower and most stressed joints are also water-proof and can be absolutely proved to be water-proof. A liberal number of drainage holes drilled so as to communicate from the down-stream side of the supposed cut-off to visible outlets at the toe of the dam might prove the condition fairly well but only after the dam were in service, when it would ordinarily be too late to use the information, if adverse.

If one could be confident of making the joints beneath the dam tight, then the doubt as to tightness of joints in the masonry above might be resolved by the use of vertical reinforcing steel on the up-stream face of the dam.

The successful design and construction of a permanently water-proof membrane is not an assured matter. The speaker has had a number of experiences in this line. One of the tunnels constructed by the Board of Water Supply of New York City became damaged by a widespread disintegration of the concrete lining where the tunnel passed through rock containing pyrite. A thin sheet steel lining to conduct off the water which filtered into the tunnel was placed inside the concrete lining, and within that a 3-ply membrane of asphalt and impregnated cotton drilling. A brick lining, 12 in. thick, was then placed within the water-proofing.

Every precaution was taken to make the membrane water-tight. Wherever it was necessary to put a grout pipe through it, interlapped fabric patches were used and compressed between flanges screwed to the grout pipes. Afterward the rock, the space between the rock and the concrete lining, that between the concrete and the water-proofing, and that between the water-proofing and the brickwork were grouted very carefully with minute attention to sequence of work and pressures. The job was successful, practically speaking, but there was plenty of evidence that water got through some of the joints or pipe holes in the water-proofing.

Recently under the speaker's direction the sloping deck of a multiple-arch dam was water-proofed, the lower parts with 3-ply, and the upper parts with 2-ply, impregnated cotton drilling and asphalt. The conditions were ideal for perfect work, the slope of the dam being convenient, the work being all in daylight, and no anchor-bolts piercing the water-proofing. A skillful and experienced superintendent was in charge and day labor was used. Again, there were some slight signs of penetration of water. It is not known to what extent the water travels laterally between the arch and the water-proofing.

There is another matter, namely, the question of freezing, which might be important in a northern climate. If water were to soak in against the water-proofing there would be a tendency for a film of water to gather between the outside jacket of masonry and the water-proofing; a gradual heaving of the outside jacket might occur progressively.

Quite apart from the saving in weight, the water-proofing, if successful, would protect the down-stream face of the dam from disintegration which too often happens, apparently owing to the freezing of seepage water on concrete that seems frequently to be all too susceptible to such damage. It was with the idea of protecting both faces of the concrete from the effect of freezing moisture that the multiple-arch dam previously mentioned was water-proofed.

The dovetail method used by Mr. Gardiner* in locking concrete together across a water-proof membrane would seem more promising than a multiplicity of bolts piercing the membrane.

An adequate factor of safety in gravity dams is most important. If water-proofing could be proved to be effective in preventing uplift pressure which it was known would otherwise actually exist, then water-proofing would have the effect either of making a lighter dam equally safe or of increasing the factor of safety for a dam of the same weight. If, however, the uplift pressure does not actually exist to an important degree in dams that are not water-proofed, the practical effect of water-proofing a dam and omitting allowance for uplift pressure would be to reduce the factor of safety which already exists in the more conservative designs. There has been more tendency among designers of dams than among designers of steel bridges, for example, to design on the "ragged edge" of safety. The factor of safety chosen for a dam will be affected, of course, in some measure according to the size of the flood that would be let loose by a failure and the number of people and value of property down stream from the dam. The failure of most dams of considerable size, however, would result in some deaths and the world can well afford better protection than has been provided in too many cases for all those obliged to dwell or to work down stream from such structures.

JEROME W. HOWE,† ASSOC. M. AM. SOC. C. E (by letter).‡—In any current consideration of this subject it is pertinent to take stock of the study devoted to it rather recently by a French Government Commission, and the conclusions respecting it embodied in the report§ to the Ministry of Public Works.

The author pays due respect to the memory of de Sazilly and Delôcre, with whom the modern rational methods of dam design and analysis orginated. Subsequently, leadership in this field of engineering passed to the United States, by virtue of the far greater extent and magnitude of masonry dam construction here, as contrasted with that in other parts of the world, even

* *Proceedings*, Am. Soc. C. E., January, 1927, Papers and Discussions, p. 112.

† Prof., Civ. Eng., Worcester Polytechnic Inst., Worcester, Mass.

‡ Received by the Secretary, January 4, 1927.

§ "Barrages de grande hauteur, résistant par leur propre poids": Instruction pour la préparation des projets et l'exécution de travaux adressée à MM. les Ingénieurs en Chef des Ponts et Chaussées par M. le Ministre des Travaux Publics (Circul. Série B, No. 67, 19 octobre 1923). Published in *Annales des Ponts et Chaussées*, Vol. 11e, Série 93e, Année No. 30.

admitting the importance and excellence of a large number of high masonry dams in Germany, Australia, and elsewhere. Latterly, however, the increase of water-power development in France has caused French engineers to focus attention again on the masonry dam, with a view to bringing about greater economy in dam construction, and thus contributing to the advance of the National water-power development. It will probably be of advantage to American engineers to keep informed of the trend of French engineering thought respecting this important subject.

The Commission of Engineers referred to did not essay any especially original approaches to its problem. It took what was probably the wiser course of studying the most advanced practice, particularly as exemplified in American dams. As a carefully considered judgment based on the whole history of the construction (and destruction) of masonry dams, and more particularly on a survey of the history of American practice, the several items of the report merit attention. In this connection, only that part of the study concerned with the matter of hydrostatic uplift is relevant. It should be remarked in passing, however, that several items of the report appear to have considerable value, particularly an appendix explaining M. Pigeaud's method of calculation, and the method of plotting results in curves of equal maximum pressure and equal maximum shear, in isostatic lines and lines of sliding.

The failure of the dam of Habra in 1881, and that of Bouzey in 1895, so shortly after the application of rational principles had been attempted for the first time in connection with the Furens Dam, caused general suspicion of the rational methods of design. Maurice Lévy, however, assigned to these failures the probable true cause—hydrostatic uplift; and he recommended interior drainage systems. He seems to have intended, rather, exterior drainage systems, for his recommendation was, in substance, to provide a protective wall separated by means of pilasters from the up-stream face. Except for possible poor bond under the influence of temperature stresses and unequal settlement, this was in effect a provision for a system of wells close behind the up-stream face, for which an under-drainage connecting canal was a necessary adjunct. Many French dams have been thus constructed in recent times.

The Commission finds that this provision has added about 30% to the volume of masonry and, consequently, to the costs, and is not favorable to a continuance of such a practice.

In its consideration of hydrostatic uplift, the Commission reviews the Italian practice, which is to calculate such uplift in most cases, and to use a unit pressure varying uniformly from $m \gamma y$ at the up-stream face to zero at the down-stream face; γ being the unit weight of water; y, the depth below water level; and m, a constant varying with conditions from one-third to unity. This constant becomes equal to zero only in low dams with most favorable conditions (especially as to foundations and materials of construction). However, the French Commission recommends the design of most masonry dams on the basis of no hydrostatic uplift.

The Commission declares itself in favor of the American practice, exemplified in the Kensico Dam, of a network of drainage wells and galleries near

the up-stream face, with perhaps a second series toward the center of the structure. In many dams the wells are too small to permit of inspection. The best practice is that adopted at Kensico, of wells 1 m. square spaced (in this instance) 85 ft. apart, with smaller wells (in this instance, six) in each interval, the latter lined with porous concrete blocks.

Respecting methods of making the up-stream face impermeable, the Commission notes the successful application of coatings in a number of instances, particularly coatings of asphaltic materials, and, as at the Elephant Butte Dam, gunite. It gives special emphasis, however, to the possibility of making the masonry of the up-stream face itself impermeable, or practically so. In this connection, also, reference is made to the Kensico Dam, which was built, as was also the Olive Bridge Dam, with a facing of pre-cast concrete blocks made with special care and placed with thin joints. The Commission's favorable reference to Kensico is well justified. A visit to the inspection galleries reveals a discharge into the gutters of the lower inspection gallery of not more than about 7 000 or 8 000 gal. per day, most of this being from the expansion joints. Very little leakage is contributed from the seepage wells.

The Commission is especially favorable to the practice of obtaining practical impermeability by enriching the cement content of the concrete for a distance back from the face varying with the water pressure. It refers to the Arrowrock and Hetch-Hetchy Dams as instances. In the former structure the mix was 1 : 9 for a thickness of from 5 to 10 ft., the remainder of the structure being poured with a 1 : 10.5 mix. At Hetch Hetchy the mix of the up-stream face was 1 : 7.5 (instead of 1 : 9 as for the mass of the structure) for a thickness of about 4 or 5 ft. Attention is called, however, to the increase in cost resulting from such a practice, and the recommendation is made that instead of securing impermeability through excess of cement over that required for strength and durability, the recent practice of the U. S. Reclamation Service of using sand cement be followed. This sand cement is obtained by grinding to a high degree of fineness a mixture of cement clinker and pebbles of hard stone (such as granite, sandstone, basalt). At the end of several years, it gives, so it is stated, a strength at least equal, sometimes even superior, to that obtained with pure cement, and it always gives a greater impermeability. Its use permits of varying the strength from bottom to top of the dam, without at the same time diminshing the imperviousness of the face. Scientific proportioning of the concrete, by the "water-cement-ratio" principle, with careful grading of the aggregates and control of the water, are also counted upon heavily to achieve a practically impervious concrete.

To prevent the under-passage of water, sheet-piling, pressure grouting of the foundation area, drainage of the foundation just down-stream from the sheet-piling, and a cut-off wall of rich concrete are recommended. Foundation drains under the up-stream part of the dam should be pierced after the injection under pressure of grout into the seams of the sub-rock. They should lead to the inspection gallery within the dam, so that their discharge can be observed. In case of excessive discharge, additional pressure injections can be effected, using for this purpose the inspection gallery and drainage wells.

At the Elephant Butte Dam 360 tons of cement were injected into the foundation rock. A double row of foundation drains was constructed. That nearest the up-stream face lies at a depth of 45 ft. below the foundation; that nearest the down-stream face is only a few feet below the foundation.

To summarize, the conclusion of this Commission was that it is usually possible by taking proper precautions (galleries, drains, surfacings, cut-off walls, pressure injections, etc.) and by meticulous care in the construction, to avoid the penetration of water under pressure either into the masonry of the dam or through the foundation rock in the vicinity of its base.

W. H. R. NIMMO,* ASSOC. M. AM. SOC. C. E. (by letter).†—The allowance to be made for uplift under masonry dams, as recommended by various designers, ranges from full hydrostatic head over the base to zero. The author assumes that for masonry dams as now constructed there will be water pressure under the base varying uniformly from two-thirds full hydrostatic head at the up-stream face to zero at the down-stream face; and he then shows that if this uplift pressure is entirely prevented by a water-tight diaphragm placed near the up-stream face the saving in material in the case of a high dam will amount to about 14 per cent.

Such a diaphragm, if adopted, would have to be placed sufficiently far back from the face of the dam to be well beyond the influence of freezing. Where uplift pressures of the intensity anticipated by the author cannot be prevented by other means a metal diaphragm may be well worth considering. For a dam on a porous foundation it is necessary to assume that the uplift pressure at the up-stream face will be equal to the full hydrostatic head, but for a well designed and carefully constructed dam on a reasonably sound rock foundation an allowance of even $\frac{2}{3}H$ seems to the writer to be excessive. If care is taken to avoid shattering of the foundation rock by the use of heavy charges of explosives, and a cut-off wall is constructed with a system of drains behind it discharging into an inspection tunnel, then only small pressures, if any, should be developed in the foundation.

The body of a high and important gravity dam should also be protected by an efficient drainage system consisting of approximately vertical drains, at frequent intervals, running in straight lines from an upper to a lower inspection gallery. All drains should be large enough to be properly cleaned. If a leak develops under the base it will be detected by the drains and can probably be stopped by boring holes and grouting under pressure. If the dam is composed of concrete a relatively richer mix may be used in the immediate vicinity of the up-stream face. If the dam is to be filled rapidly it may be advisable to plaster the water face with a cement gun, since, although this coating may not be permanent, the concrete is thereby protected from intrusion of water until it has become thoroughly seasoned. If these precautions are taken, only a small allowance for uplift should be necessary.

* Chf. Draftsman, Main Roads Comm., Brisbane, Queensland, Australia.
† Received by the Secretary, January 13, 1927.

A commission of French engineers, reporting on the design and construction of high gravity dams, expressed the opinion that it is usually possible to prevent the penetration of water under pressure either into the body of the dam itself or into the foundation; and that, except under special circumstances, uplift should not be taken into account in designing a gravity dam.*

An arched dam, recently built across the Jogne River, in Switzerland, and having a height of approximately 195 ft., was provided with drains from the foundation. Regular observation after the filling of the reservoir failed to disclose any flow of water from the drains.† If it be conceded that the author's two-thirds rule should be generally adopted, an arched dam the stability of which is not menaced by uplift pressure has a great advantage over a gravity dam in this respect.

E. W. LANE,‡ M. AM. SOC. C. E. (by letter).§—There seems to be enough saving in cost in the method of dam construction proposed by Mr. Pagon to make it worthy of serious consideration in some cases, but it should be used with caution. Any reduction in the volume of masonry in a dam means a decrease in weight and, therefore, a smaller factor of safety against sliding. In many cases the danger of sliding is much greater than that of over-turning even with uplift neglected.

The writer does not recall a single instance of a dam which had a resultant pressure within the middle third, neglecting uplift, and failed from overturning (with the possible exception of cases due to vacuum effects in over-fall dams); but several with such a section, or even a heavier one, have failed from sliding. At many sites the requirements for stability against sliding, therefore, may preclude the use of Mr. Pagon's suggestion.

The necessity of considering uplift in the bed-rock is important even if it is considered in the design of the cross-section of the dam, especially if the foundation is a rock with strata horizontal or dipping slightly down stream. In such cases the writer believes the foundation layers should be grouted or drained or, better still, both; although in some cases the cut-off with a diaphragm as suggested by Mr. Pagon might be preferable.

Unless the seepage is cut off, or vented, to prevent the building up of pressures in the bed-rock, the usual allowance for upward pressure may be insufficient. Consider a dam founded on an impervious rock layer, below which is a seam; for some distance down stream from the dam the rock surface slopes away, as in Fig. 5 (*a*). If the water stored above the dam enters the seam near the dam, and, decreasing uniformly in pressure, escapes at the lower end of the seam, the uplift on the dam and layer may be so much greater than that usually assumed in dam design that the factor of safety against sliding, even considering the extra weight of the rock layer, is much less than that estimated. Such a condition may have caused the sliding of the dams at Austin, Tex., and Austin, Pa. In case the strata slope down, as

* Ministère des Travaux Publics, Circulaire Série B, No. 67, Paris, October 19, 1923.

† "La Construction des Barrages-Voutes en Suisse," by H. E. Gruner, *Le Genie Civil*, August 29, 1925.

‡ Hydr. Engr., Mississippi River Spillway Board, U. S. War Dept., New Orleans, La.

§ Received by the Secretary, January 24, 1927.

in Fig. 5 (*b*), the pervious joint at which the water escapes may be so far from the dam that practically the full upward pressure is effective over the entire base, although the weight of the rock between the seam and the bottom of the dam would partly offset this. A dam with such a foundation could not readily slide, but the factor of safety against over-turning would be reduced.

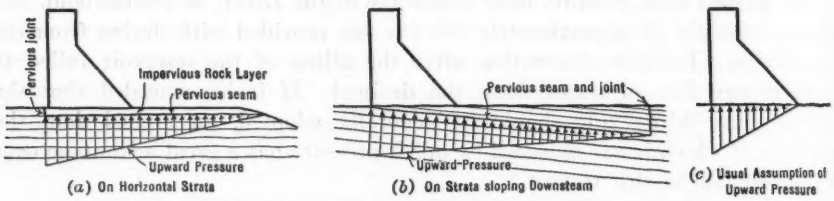

FIG. 5.—VARIOUS CONDITIONS OF UPWARD PRESSURE UNDER DAMS.

For conditions which permit the use of the method proposed by Mr. Pagon the same result as regards stability of the dam proper against over-turning could be more cheaply accomplished in a concrete dam by adding tension reinforcement near the up-stream face to resist the over-turning effect of the upward pressure. It would be necessary to take precautions to have the rods well protected against corrosion at the horizontal construction points, but this could be easily done. If necessary, the tension rods could be encased in a precast unit before being placed in the dam. Many dams are now built in which the entire reliance against failure is placed on the protection of a few inches of concrete, and it should not seem very radical, therefore, when five or ten times that thickness is used to insure an effect which may not be necessary for stability. The writer does not wish to advocate this form of construction as he has not given the subject sufficient study, but the possibility would seem worth consideration. This method, however, would not prevent seepage through the dam as would a diaphragm.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

OSWALD HERBERT ERNST, M. Am. Soc. C. E.*

DIED MARCH 21, 1926.

Oswald Herbert Ernst was born at Cincinnati, Ohio, on June 27, 1842. He was the son of Andrew Henry and Sarah (Otis) Ernst, his mother having been a daughter of George Alexander Otis of the distinguished Massachusetts family of that name. He entered Harvard College in 1858 at the age of sixteen, his education up to that time having been received at private schools. After two years at Harvard he received an appointment to the West Point Military Academy from the State of Ohio, and was admitted to that institution on July 1, 1860. He was graduated from West Point on June 13, 1864, and was promoted on that date to the rank of First Lieutenant, Corps of Engineers, U. S. Army.

Mr. Ernst's career at West Point was marked not only by high grade in academic studies but by characteristics of personal prominence in other respects. This is shown by the fact that during his last three years of cadet life he was selected to fill responsible positions in the military organization of the Cadet Corps. In his second year at West Point he was appointed one of the ranking Corporals; in his third year he was the First Sergeant of one of the four Cadet Companies; and during his entire fourth year he was First Captain of the Cadet Battalion.

The selection of cadets to fill the positions of officers in the Battalion, is made by the Academy authorities with the intent of securing those cadets most likely to inspire the confidence and command the respect of their cadet associates The position of First Captain in the Corps at that time, and until quite recently, was the most responsible one entrusted to a cadet. The selection of the First Captain is based partly on soldierly bearing and partly on the confidence of the authorities in his ability, his good judgment, his impartial disposition, and his appreciation of the many responsibilities that devolve upon him. Mr. Ernst met all these requirements and expectations to the satisfaction of the authorities and with such calm, courteous, and impressive dignity that his associates dubbed him "Deacon" Ernst, a title which colleagues of cadet days ever after applied to him.

For the first fourteen years after his graduation he was engaged almost entirely on duty purely of a military nature. During the first year after graduation, as an Assistant Engineer of the Army of Tennessee, he was engaged in the battle and siege of Atlanta, and other active field operations. His other purely military occupation in these years embraced construction work on fortifications in the Harbor of San Francisco, Calif. For

* Memoir prepared by S. E. Tillman, Brig. Gen., U. S. A. (*Retired*), New York, N. Y.

three years, from 1868 to 1871, he was an Instructor in the Engineering School of Practice at Willets Point, Long Island; and for seven years, from 1871 to 1878, he served as Instructor in Practical Engineering at the West Point Military Academy. During this latter period he was assigned the civil detail of Astronomer of the United States Commission to observe the solar eclipse in Spain in December, 1870.

For the next fifteen years, 1878 to 1893, Captain (later, Major) Ernst's services were devoted to engineering and other works of a civil nature. These duties included improvements in the navigation of the Osage River, Missouri, and of the Mississippi and Missouri Rivers above their point of junction, at various places in Illinois and Missouri. The improvements extended also to the interstate harbors of these rivers. During part of this period, from 1886 to 1889, he was also in charge of the river and harbor improvements of Texas and from 1889 to 1893, of the Public Grounds and Buildings of Washington, D. C.

Captain Ernst received the brevet of Captain in March, 1865, for "faithful and meritorious service", within less than a year after his graduation. He attained the rank of Captain of Engineers in 1867; that of Major in 1882; and Lieutenant Colonel in 1895. In the period between 1881 and 1889 he was a member of the Mississippi River Commission and of various engineering boards, on river and harbor works and bridge construction. He reached the rank of Colonel of Engineers in 1903.

From 1893 to 1898, Major (later Lieutenant Colonel) Ernst was in command at West Point and Superintendent of the Military Academy. In 1898 he left West Point to take part in the Spanish-American War, and commanded the First Brigade of the First Division of the First Corps in the Porto Rican Campaign in 1898 and 1899. When the war was over, he returned to engineering work and from 1899 to 1904, was member of a Commission to determine a route for a canal across Panama. From 1903 to 1906 he was President of the Mississippi River Commission and also from 1902 to 1905, Chairman of the Board of Engineers to survey a route for a 14-ft. waterway between Chicago, Ill., and St. Louis, Mo. During 1905 and 1906 he served as a member of the Isthmian Canal Commission. Colonel Ernst was retired from active service with the rank of Brigadier General on June 27, 1906, having reached the legal age of retirement.

After retirement he continued engineering work as a member of the International Waterways Commission until 1913. He was appointed to this body in 1903 and was thereafter Chairman of the American Section. From 1906 to 1908 he was Consulting Engineer of the East Side Levee Association for the protection of the American bottom opposite St. Louis, and prepared the plans therefor. In 1909 he was Consulting Engineer on plans for the drainage of the Coldwater-Tallahatchie Basin in the alluvial bottom of the Mississippi River. In 1914 and 1915 he was President of the Board of Consulting Engineers convened by the Miami Conservancy District to examine plans to protect the valley of the Miami River, Ohio, from the destructive effects of floods. In November, 1916, he was promoted to the rank of Major General on the retired list of the Army.

The personal characteristics of General Ernst were so fine and rare as to deserve special mention. He was of large and impressive physique, his manner was courteous and friendly, rather gentle but always dignified; his physiognomy, especially his eyes, and his calm, careful expression in conversation inspired ready confidence and brought conviction of his honest and frank nature. His mind was not rapid in operation, but extremely logical and sure in conclusion; his sense of and desire for justice were strong and his integrity was of the loftiest; his ideals of conduct and his sense of duty in all relations of life were very high and these he endeavored to inculcate by precept as well as example. His life is an illustration of beautiful conformity to the motto of his Alma Mater—Duty, Honor, Country—which motto he helped to frame. His record is one of long, honorable, distinguished, and useful service to his country.

He died on March 21, 1922, in his eighty-fourth year. His widow and two daughters, Helen Ernst and Mrs. William Morton Grinnell, survive him.

General Ernst was elected a Member of the American Society of Civil Engineers on July 4, 1888.

HERBERT CLARK FELTON, M. Am. Soc. C. E.*

DIED MAY 15, 1926.

Herbert Clark Felton, the son of Amory and Nancy Perkins (Boynton) Felton, was born in Troy, N. Y., on November 3, 1846. After completing his early education in the local schools he entered Rensselaer Polytechnic Institute, from which he was graduated in 1866 with the degree of Civil Engineer.

In the fall of that year, Mr. Felton entered the employ of the Novelty Iron Works, in New York, N. Y., as Draftsman, which connection he continued for two years. During this same period he taught Mechanical Drawing in the School of Science at Cooper Union Free Night School.

Late in 1868, he accepted an offer with Messrs. Brown Brothers and Company, Bankers, of New York, to undertake preliminary and location surveys for a number of railroad projects in which that Company was interested. That his services were satisfactory to his employers is attested by the fact that he remained with them for a period of twenty years. During the early part of his assignment, he was Superintendent of the Fort Lee Railroad and made a survey for a new railroad line along the west bank of the Hudson River, starting opposite New York. In 1872-73 he made a location survey for the proposed Camden and Manchester Railroad, extending from Camden to Manchester (now Lakehurst), N. J. Had it been built, this railroad would have given the New Jersey Southern (Central Railroad of New Jersey) entrance to Philadelphia, Pa.

On December 2, 1873, the Kaighn Point, Camden, and Philadelphia Ferry Company was purchased by Brown Brothers and Company, and Mr. Felton

* Memoir prepared by Henry J. Sherman, M. Am. Soc. C. E.

assumed the duties of Secretary, Treasurer, and Superintendent of this Company which operated ferry boats between Chestnut and South Streets, Philadelphia, and Kaighn Point, Camden. In 1888, the Reading Railroad purchased this Ferry Company from Brown Brothers and Company, retaining Mr. Felton in the capacity of Superintendent, which position he held until his retirement on February 1, 1917.

Mr. Felton was married on April 10, 1872, to Harriet Louisa Job, of Troy, N. Y., who survives him.

He was one of the organizers of the Camden National Bank, with offices in Camden and Philadelphia, having been a Director from the time it opened its doors for business on August 13, 1885. On January 13, 1905, he was elected Vice-President and continued in that capacity until his death. Mr. Felton was also a Director of the Atlantic City Railroad Company, the Port Reading Railroad Company, and, at the time of his death, was a member of the Chamber of Commerce of Camden and the Chamber of Commerce of Philadelphia.

He was an exceedingly jovial and kindly man, much beloved by all who knew him.

Mr. Felton was elected a Member of the American Society of Civil Engineers on June 1, 1887. He was also a member of the Society of Naval Architects and Marine Engineers.

SAMUEL BROWNLEE FISHER, M. Am. Soc. C. E.*

DIED JULY 9, 1926.

Samuel Brownlee Fisher, the son of Jacob Piper and Jane Thompson (Brownlee) Fisher, was born at Cherry Fork, Ohio, on October 22, 1846. He was a student at Washington and Jefferson College, Washington, Pa., from 1862 to 1868. His mental attainments brought him recognition from his Alma Mater and he received the degrees of Bachelor of Science in 1868, Master of Science in 1871, and Doctor of Science in 1908.

From 1869 to 1871 he was in the Rogers Locomotive and Machine Works, at Paterson, N. J., learning locomotive building. During the following year he was engaged in the locomotive repair shops of the Pittsburgh, Cincinnati, and St. Louis Railway Company, at Dennison, Ohio, and from 1872 to 1873, he held a position with the Pittsburgh Locomotive Works, at Allegheny, Pa.

From 1873 to 1886 Mr. Fisher was employed in the Engineer's Office of the Pennsylvania Company at Pittsburgh, Pa., and was engaged in various work, including preliminary and location surveys in Western Pennsylvania and Northeastern Ohio, and on second-track work. He was also engaged on the maintenance and construction of harbors on Lake Erie, sidings, the relocation of the Pittsburgh, Fort Wayne, and Chicago Railway and its branch lines, and on the construction of a low-grade line along Beaver River from New Brighton to New Castle, Pa.

* Memoir prepared by A. W. Galbreath, M. Am. Soc. C. E.

From 1886 to 1890 he was Chief Engineer of the Milwaukee and Northern Railway Company at Green Bay, Wis. He surveyed and built the railway from Iron Mountain to Republic, and from Rockland to Sidnaw, in Michigan. He also superintended the location of the Green Bay Kewaunee Railway.

From 1890 to 1892 he was in the employ of the Minneapolis, St. Paul, and Sault Ste. Marie Railway Company and built the Hankinson Branch of that road. He also made a reconnaissance survey to the International Boundary.

Mr. Fisher was Chief Engineer of the Everett and Monte Cristo Railway from 1892 to 1894 and built a line from Puget Sound to the heart of the Cascade Mountain Range.

In the winter of 1894-95 he made a reconnaissance survey for the railway from the St. Croix River near St. Paul, Minn., to Ashland, Wis. Mr. Fisher was also engaged on construction work for contractors on a timber branch of the Chicago, Milwaukee, and St. Paul Railway among the Wisconsin lakes.

During 1895 and 1896 he served as Chief Engineer of the St. Louis and Kansas City Railway, and as Assistant Chief Engineer of the Missouri, Kansas, and Texas Railway. From 1896 to 1913 he held the position of Chief Engineer of the Missouri, Kansas and Texas Railway, with headquarters at St. Louis, Mo. From 1913 to 1915 Mr. Fisher was Chairman of the Valuation Committee, at St. Louis, and from 1915 until his death, he served as Consulting Engineer of the Valuation Department of this railroad, with headquarters in Parsons, Kans. The many improvements, extensions, and the excellent condition of the Missouri-Kansas-Texas System at the present time are due in a large measure to his work.

During his railroad career Mr. Fisher made countless friends in both business and railroad circles. He was a man of unusual ability and foresight and his retentive mind was a source of information that was considered reliable by his employers and the leaders in his profession. He was a born student and was continually adding to his intimate knowledge of a wide range of subjects of which those pertaining to his life's work were only a small part. At the age when most people consider their education finished, he was investigating new fields of knowledge. When sixty-nine years of age he attended night sessions in the Parsons High School to add to his knowledge the Spanish language. He was also deeply interested in religion.

While the Engineering Profession and Mr. Fisher's large circle of associates and friends have suffered a great loss in his passing, his sterling character, capacity, vision, and the example of his Christian life will be a continual source of inspiration to efforts for more useful and helpful lives.

He died at Parsons, on July 9, 1926, following an illness of a few days due to heat prostration. He was married on February 7, 1882, to Agnes Crooks, who died in December, 1906. He is survived by a son, Brownlee Fisher, of California, a daughter, Anne Fisher, of De Soto, Mo., and a grandson.

He was a member and an Elder of the Second Presbyterian Church of St. Louis for many years. He was also a Charter Member of the American

Railway Engineering Association, and a member of the Engineers Club of St. Louis and of the St. Louis Railway Club.

Mr. Fisher was elected a Member of the American Society of Civil Engineers on October 7, 1903.

HARRY ADOLPH OESTERLE, JR., Assoc. M. Am. Soc. C. E.*

DIED MAY 24, 1926.

Harry Adolph Oesterle, Jr., the son of Henry and Margaret (Weber) Oesterle, was born in Newark, N. J., on December 23, 1891. Following his primary education he entered the Newark Technical School where he studied for four years, supplementing this work by special courses at the Brooklyn Polytechnic Institute, Brooklyn, N. Y.

During 1913, Mr. Oesterle was connected with the Research Corporation of New York, N. Y., on the design of electrical precipitators and, later, with the Guarantee Construction Company on general plant construction and ash-handling systems.

From June, 1914, until March, 1915, he held the position of Designer with The Balbach Smelting and Refining Company, of Newark, N. J., on the design and construction of an electrical precipitator for smelter flue gases.

In March, 1915, Mr. Oesterle accepted a position with The Westinghouse, Church, Kerr, and Company, of New York, as Designing Draftsman and, later, was promoted to the position of Junior Engineer. His work consisted principally of power-house and sugar refinery design.

From December, 1916, until his death he served as Resident Engineer of the New Niquero Sugar Company, of Central Niquero, Cuba, in complete charge of all field work, including railroad construction and maintenance, as well as surveying and general design and construction. It was while employed by the New Niquero Sugar Company that Mr. Oesterle met his death as the result of a premature dynamite explosion.

He was married on March 14, 1918, to Sarah T. Maher, of Newark, N. J., who with a daughter, Sarah Adelaide, survives him.

He was an Associate Member of the American Society of Mechanical Engineers and a Member of the American National Engineering Societies of Cuba.

Mr. Oesterle was elected an Associate Member of the American Society of Civil Engineers on July 6, 1925.

* Memoir compiled from information on file at the Headquarters of the Society.